$N=2$ Supergravity Lagrangian Coupled to Tensor Multiplets with Electric and Magnetic Fluxes

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Abstract

We derive the full $N=2$ supergravity Lagrangian which contains a symplectic invariant scalar potential in terms of electric and magnetic charges. As shown in reference [1], the appearance of magnetic charges is allowed only if tensor multiplets are present and a suitable Fayet–Iliopoulos term is included in the fermion transformation laws. We generalize the procedure in the quoted reference by adding further a Fayet–Iliopoulos term which allows the introduction of electric charges in such a way that the potential and the equations of motion of the theory are symplectic invariant. The theory is further generalized to include an ordinary electric gauging and the form of the resulting scalar potential is given.

1 Introduction

Standard $D = 4$ supergravities with N supersymmetries are usually described, for $N \leq 4$, as the coupling of the gravitational multiplet to matter multiplets whose bosonic content is limited to scalar or vector fields. Multiplets containing massless twofold–antisymmetric tensor fields are usually converted into ordinary vector or scalar multiplets using Hodge duality in four dimensions between tensors and scalars [2]. However the description of the couplings and the geometry of the σ–model is quite different in the dual theory. This is the case, for example, of the coupling of an arbitrary number of tensor multiplets (linear multiplets) in $N = 1$ Supergravity [3] and of the coupling of the scalar–tensor multiplet in $N = 2$ supergravity [4]. From the point of view of compactification of ten dimensional supergravities down to four dimensions, tensor multiplets appear naturally in $D = 4$ due to the presence of antisymmetric tensors in the $D = 10$ theory. For a full understanding of four dimensional gauged or ungauged supergravities in the presence of tensor field fluxes in the parent ten dimensional theories, it seems appropriate to keep at least some of the massive tensor fields undualized in the resulting four dimensional Lagrangian.
In reference [5] it has been shown, at a purely bosonic level, that ten dimensional Type IIA and Type IIB supergravities compactified on a Calabi–Yau threefold, give rise to theories with with tensor multiplets and that in the presence of general fluxes they contain both electric and magnetic charges. Particularly, some of the tensors might become massive due to the presence of the magnetic fluxes. For instance in the Type IIB case the universal hypermultiplet appears as a double tensor multiplet $(B^{(1)}_{\mu\nu}, B^{(2)}_{\mu\nu}, \ell, \phi)$ [6] and one of the two tensors, which come either from the R–R and NS–NS 3–form respectively, is massive when the magnetic R–R or NS–NS flux is switched on, due to following coupling with the vector field strengths:

$$\mathcal{F}^\Lambda \to \hat{\mathcal{F}}^\Lambda = \mathcal{F}^\Lambda + 2m^{I\Lambda} B_I \tag{1.1}$$

where $I = 1$ or $I = 2$ according to the case. Furthermore the theory with both electric and magnetic fluxes has a symplectic invariant scalar potential and the set of equations of motion and Bianchi identities for the vector field strengths are also symplectic invariant as in the ungauged supergravity, provided that the electric and magnetic fluxes ($m^{I\Lambda}$, e^I_Λ) transform as a symplectic vector. In reference [7], the scalar potential in the presence of both

R–R and NS–NS fluxes was derived.
This result naturally rises the question of how to understand the magnetic charges from a purely four dimensional point of view, without introducing ill–defined magnetic gauge potentials in the theory, as it was done in reference [8]. The answer was found in reference [1] where it was shown that in $N = 2$ supergravity coupled to vector multiplets and to the $N = 2$ scalar–tensor multiplet (tensor multiplet in the following), the appearance of magnetic charges can be understood as the consequence of the redefinition of the vector field-strengths as in equation (1.1), where now, however, the index I runs on the number of antisymmetric tensor fields contained in the tensor multiplet. This is consistent with supersymmetry provided one performs a non–trivial generalization of the fermionic transformation laws of the ungauged theory. This modification amounts to adding to the transformation laws of the fermions a magnetic "mass–shift" (or Fayet–Iliopoulos term) which is therefore allowed if and only if antisymmetric tensor fields are present.
In the quoted reference the theory was also generalized performing an ordinary electric gauging. However the electric charges introduced by the gauging cannot be identified with the electric charges of references [5],[7] since they are not naturally paired with the magnetic charges giving rise to a symplectic vector. Indeed, if one thinks of $N = 2$ supergravity with tensor multiplets as coming from the dualization of the axionic fields on the quaternionic manifold $\mathcal{M}_Q$ then one is left with a new manifold $\mathcal{M}_T$ without the translational symmetries which have been dualized. Therefore one can not obtain, on the manifold $\mathcal{M}_T$, electric Killing vectors which are charged under the original axionic symmetries.
In reference [9], the dualization of the axionic coordinates is performed after the translational gauging has been implemented. This way one obtains that the electric charges e^I_Λ associated to the gauging are naturally paired with the magnetic charges which one can generate by means of a suitable symplectic rotation. The approach of [9] however is not the most efficient in order to implement supersymmetry and to derive the supersymmetric Lagrangian, which in fact was not constructed.
In this paper we take the attitude of constructing the full supersymmetric theory, namely the Lagrangian and its supersymmetry transformation laws, directly from the content of the gravitational, vector, and tensor supermultiplets, thus using as a σ–model the manifold $\mathcal{M}_T \otimes \mathcal{M}_{SK}$. Here $\mathcal{M}_T$ is the σ–model parametrized by the scalar fields of the $N = 2$ tensor multiplet, and $\mathcal{M}_{SK}$ is the Special Kähler manifold of the vector multiplets (which re-

mains untouched with respect to the standard $N=2$ supergravity). Our results show that it is possible to have electric charges naturally paired with the magnetic ones without performing any translational gauging, but simply adding, to the transformation laws of the fermions, a further Fayet–Iliopoulos term or "electric mass–shift", besides the magnetic one of reference [1]. As a result of our analysis of the Bianchi identities in superspace, supersymmetry then implies that the mass–shifts are symplectic invariant if one assumes that the vector $\left(m^{I\Lambda}, e^I_\Lambda\right)$ is symplectic, that is transforms as the symplectic section of the Special Geometry. Since the scalar potential is constructed as a quadratic form in the mass–shifts it is automatically symplectic invariant. As a particular case the scalar potential obtained in this way coincides with the scalar potential obtained in the case of Calabi–Yau compactification from Type IIA and Type IIB supergravity [7], when the scalar–tensor multiplets are suitably specified. Therefore, as a particular case of our construction we obtain the complete four dimensional supergravity corresponding to Type IIB compactification on CY in the presence of both R–R and NS–NS electric and magnetic fluxes.

Note that as in the standard ungauged $N=2$ theory the equations of motion and the Bianchi identities for the vectors are still covariant under symplectic transformations of the electric and magnetic field–strengths. If we perform an ordinary gauging of the theory, then of course the on–shell symplectic invariance is broken leaving however the symplectic invariance in the sector parametrized by $m^{I\Lambda}, e^I_\Lambda$.

We also note that our Lagrangian is invariant under the combined gauge transformations [5]:

$$\begin{aligned} \delta B_{I\mu\nu} &= \partial_{[\mu}\Lambda_{I\nu]} &(1.2)\\ \delta A^\Lambda_\mu &= -2m^{I\Lambda}\Lambda_{I\mu} &(1.3)\end{aligned}$$

where I is the number of 2–forms of the tensor multiplet, and $\Lambda_{I\mu}$ is a 1–form gauge parameter, if and only if the electric and magnetic charges satisfy the constraint:

$$e^I_\Lambda m^{J\Lambda} - e^J_\Lambda m^{I\Lambda} = 0. \tag{1.4}$$

The same constraint also appears from the supersymmetry Ward identity for the scalar potential [9]. This constraint is a generalization of the tadpole cancellation mechanism in ten dimensions, where the indices I, J take now

only the values $1, 2$ associated to the NS-NS and R-R 2–form.

The paper is organized as follows:
In Section 2 we describe how to generalize the Fayet–Iliopoulos mechanism related to magnetic charges to include also a further mass–shift related to the electric charges, therefore obtaining symplectic invariant expression for the mass–shifts of the fermions.
In Section 3 we give the Lagrangian and the supersymmetry transformation laws for the $N = 2$ supergravity coupled to an arbitrary number of vector multiplets and to the tensor multiplet in our general setting, where electric and magnetic charges are present (the method we use is described in Appendix B).
In Subsection 3.1 we further generalize the Lagrangian by gauging the electric vector potentials with an arbitrary group $\mathcal{G}$, while in Subsection 3.2 we discuss the general form of the scalar potential in this more general setting.
In Appendix A we give the completion of the Lagrangian and supersymmetry transformation laws including 4–fermion and 3–fermion terms respectively which are not present in the main text.
In Appendix B we present the superspace approach for the solution of the Bianchi identities and the rheonomic approach for the construction of the Lagrangian.

2 The $D = 4$ $N = 2$ Supergravity theory coupled to vector multiplets and scalar–tensor multiplets

In reference [1], [4] the general ungauged $N = 2$ theory in the presence of vector multiplets and scalar–tensor multiplets was constructed. We recall the basic definitions. In this theory we have besides the gravitational multiplet, n_V vector multiplets and n_H hypermultiplets. The gravitational multiplet contains the graviton V^a, the (anti)–chiral gravitinos (ψ^A), ψ_A and the graviphoton A^0_μ, where $A = 1, 2$ is the SU(2) R–symmetry index. The n_V vector multiplets contain n_V vector, $2n_V$ (anti)–chiral gauginos $(\lambda^{\bar{\imath}}_A)$, λ^{iA} and the complex scalar fields z^i, $i = 1, \ldots, n_V$ which parametrize a $2n_V$ dimensional special Kähler manifold $\mathcal{M}_{SK}$. We denote by A^Λ_μ, $\Lambda = 0, \ldots, n_V$, the graviphoton ($\Lambda = 0$) and the n_V vectors of the vector multiplets, where

Λ is the symplectic index in the half upper part of the holomorphic section of the symplectic bundle which defines the special Kähler manifold $\mathcal{M}_{SK}$. To this sector we add a scalar–tensor multiplet (in the following tensor multiplet) whose content is: $(B_{I\,\mu\nu}, \zeta_\alpha, \zeta^\alpha, q^u)$ $I = 1, \dots, n_T$, $u = 1, \dots, 4n_H - n_T$, $\alpha = 1, \dots, 2n_H$ where (ζ^α) ζ_α are the (anti)–chiral component of the spin 1/2 fermions. This tensor multiplet can be understood as resulting from the hypermultiplet sector of standard $N = 2$ supergravity[1] by dualization of the axionic coordinates of the quaternionic manifold. Indeed, the n_H hypermultiplets, in the standard formulation contain $2n_H$ (anti)–chiral hyperinos (ζ^α), ζ_α, where $\alpha = 1, \dots, 2n_H$ is a $\mathrm{Sp}(2n_H, \mathbb{R})$ index, and $q^{\hat{u}}$ scalars, $\hat{u} = 1, \dots, 4n_H$, which parametrize a $4n_H$–dimensional quaternionic manifold $\mathcal{M}_Q$. If the quaternionic manifold $\mathcal{M}_Q$ admits n_T translational isometries, we can split the scalars $q^{\hat{u}} = (q^u, q^I)$, $u = 1, \dots, 4n_H - n_T$, $I = 1, \dots, n_T$, where the q^I's are the axionic scalars associated to the translations that can therefore be dualized into n_T tensors $B_{I\mu\nu}$.

The σ–model of the resulting theory is parametrized by the coordinates q^u and in the following we will refer to it as $\mathcal{M}_T$. While in the standard $N = 2$ supergravity the quaternionic manifold has holonomy contained in $\mathrm{SU}(2)\otimes\mathrm{Sp}(2n_H, \mathbb{R})$, $\mathcal{M}_T$ has a reduced holonomy which is however contained in $\mathrm{SU}(2)\otimes\mathrm{Sp}(2n_H, \mathbb{R})\otimes\mathrm{SO}(n_T)$.

In reference [1] it was found that $N = 2$ supergravity coupled to n_V vector multiplets and a tensor multiplet admits a non–trivial extension given by the addition of Fayet–Iliopoulos terms (or magnetic mass–shifts) which generate a non–trivial scalar potential. Indeed, in order to satisfy the requirements of supersymmetry, one finds, by suitable analysis of superspace Bianchi identities, that the transformation laws of the fermions may be extended with mass–shifts (Fayet–Iliopoulos terms) defined by:

$$\delta\psi_{A\mu} = \delta_0\psi_{A\mu} + \mathrm{i}S^{(m)}_{AB}\gamma_\mu\epsilon^B \tag{2.1}$$
$$\delta\lambda^{iA} = \delta_0\lambda^{iA} + W^{iAB(m)}\epsilon_B \tag{2.2}$$
$$\delta\zeta_\alpha = \delta_0\zeta_\alpha + N^{A(m)}_\alpha\epsilon_A \tag{2.3}$$

where the explicit form of the δ_0–shifts can be read from equations (3.12, 3.13,3.14) of section 3 up to 3–fermion terms (the 3–fermion terms are given

[1] Here and in the following by standard $N = 2$ supergravity we mean $N = 2$ supergravity coupled to n_V vector multiplets and n_H hypermultiplets as formulated in references [10],[11].

in Appendix A) and the "magnetic" mass–shifts are given by:

$$S^{(m)}_{AB} = -\frac{\mathrm{i}}{2}\sigma^x_{AB}\omega^x_I m^{I\Lambda} M_\Lambda \tag{2.4}$$
$$W^{iAB(m)} = -\mathrm{i}g^{i\bar{\jmath}}\sigma^{x\,AB}\omega^x_I m^{I\Lambda}\overline{h}_{\bar{\jmath}\Lambda} \tag{2.5}$$
$$N^{A(m)}_\alpha = -2\mathcal{U}^A_{I\,\alpha} m^{I\Lambda} M_\Lambda \tag{2.6}$$

where $m^{I\Lambda}$, $I = 1, \ldots, n_T$ and $\Lambda = 0, \ldots, n_V$, are constants with the dimension of a mass and as we will see in the following, they can be interpreted as magnetic charges. Furthermore $\omega^x_I(q^u)$ and $\mathcal{U}_I{}^{A\alpha}(q^u)$ are SO(n_T) vector–valued fields on $\mathcal{M}_T$ carrying additional indices in the adjoint of SU(2) and in fundamental of SU(2) $\otimes$ Sp($2n, \mathbb{R}$), respectively. Finally $M(z, \overline{z})_\Lambda$ and $h(z, \overline{z})^j_\Lambda$ are the lower component of the symplectic sections

$$V = (L^\Lambda, M_\Lambda) \tag{2.7}$$
$$U_i = D_i V = (f^\Lambda_i, h_{\Lambda i}) \tag{2.8}$$

of the special geometry, which transform covariantly under the $(2n_V + 2) \times (2n_V + 2)$ symplectic matrix:

$$S = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{2.9}$$

where

$$A^T C - C^T A = B^T D - D^T B = 0, \quad A^T D - C^T B = \mathbb{1} \tag{2.10}$$

It is important to note that, in order to have a supersymmetric theory, the magnetic mass–shift deformations imply that the vector field–strengths have to be redefined as follows [5],[12],[13]:

$$\mathcal{F}^\Lambda \to \hat{\mathcal{F}}^\Lambda = \mathcal{F}^\Lambda + 2m^{I\Lambda} B_I. \tag{2.11}$$

From equation (1.1) it follows that the resulting theory must contain massive tensor fields $B_{I\mu\nu}$ since by a Higgs mechanism a subset (for $n_T < n_V + 1$) of the vector potentials are eaten giving mass to the tensors.
Given the mass–shifts, the scalar potential can be computed from the usual Ward identity of supersymmetry and turns out to be:

$$\begin{aligned}\mathcal{V} &= -\frac{1}{2}\omega^x_I\omega^x_J\left(\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}+\left(\mathrm{Re}\,\mathcal{N}\mathrm{Im}\mathcal{N}^{-1}\,\mathrm{Re}\mathcal{N}\right)_{\Lambda\Sigma}\right)m^{I\Lambda}m^{J\Sigma}+\\ &\quad +4m^{I\Lambda}m^{J\Sigma}\overline{M}_\Lambda M_\Sigma\left(\mathcal{M}_{IJ}-\omega^x_I\omega^x_J\right) \end{aligned} \tag{2.12}$$

where $\mathcal{N}_{\Lambda\Sigma}$ is the period matrix of the special geometry, and the dependence on the coordinates of $\mathcal{M}_T$ appears only in $\omega^x_I(q)$.
If one thinks of this supergravity theory as coming from standard $N=2$ supergravity where dualization of the axionic coordinates of the quaternionic manifold has been performed, one can easily see that ω^x_I and $\mathcal{U}_I{}^{A\alpha}$ are the remnants of the SU(2) connection and vielbeins of the quaternionic manifold in the directions I of the dualized coordinates q^I [1]. Moreover the mass parameters $m^{I\Lambda}$ can be associated to the dual algebra of the translation group isometries on the quaternionic manifold and could be interpreted as magnetic charges allowed by the presence of the tensor multiplets [1], [9].
In fact the potential (2.12), in the case of Calabi–Yau compactification of type IIB string theory [9], appears to be the magnetic part of the symplectic invariant scalar potentials containing, besides the magnetic charges $m^{I\Lambda}$ $I=1,2$, also the Abelian electric charges e^I_Λ. This raises the question whether we can further modify the shifts in the fermionic transformation laws in such a way that the resulting potentials contain both electric and magnetic charges in a symplectic invariant setting.
We now show that it is actually possible to add further Fayet–Iliopoulos "electric" mass–shifts to the fermion transformation laws which contain electric charges, in such a way that the total shifts are symplectic invariant, thereby giving rise to a symplectic invariant scalar potential. The technical procedure to arrive to such an extension is explained in detail in the Appendix B. There we show that, using Bianchi identities in superspace, the global Fayet–Iliopoulos term appearing in the transformation laws of the fermions can be written in the following symplectic invariant way:

$$S_{AB}\equiv S^{(e)}_{AB}+S^{(m)}_{AB} = \frac{\mathrm{i}}{2}\sigma^x_{AB}\,\omega^x_I\left(e^I_\Lambda L^\Lambda - m^{I\Lambda}M_\Lambda\right) \tag{2.13}$$

$$W^{iAB}\equiv W^{iAB(e)}+W^{iAB(m)} = \mathrm{i}g^{i\bar{\jmath}}\sigma^{x\,AB}\omega^x_I\left(e^I_\Lambda\overline{f}^\Lambda_{\bar{\jmath}}-m^{I\Lambda}\overline{h}_{\bar{\jmath}\Lambda}\right) \tag{2.14}$$

$$N^A_\alpha\equiv N^{A(e)}_\alpha+N^{A(m)}_\alpha = 2\mathcal{U}_I{}^A{}_\alpha\left(e^I_\Lambda L^\Lambda - m^{I\Lambda}M_\Lambda\right) \tag{2.15}$$

where we have assumed that the vector $(m^{I\Lambda}, e^I_\Lambda)$ transforms as the symplectic section V of equation (2.7). In this case the scalar potential turns out to

be symplectic invariant and its explicit form is given by:

$$\begin{aligned}\mathcal{V} &= 4\left(\mathcal{M}_{IJ}-\omega^x_I\omega^x_J\right)\left(m^{I\Lambda}\overline{M}_\Lambda - e^I_\Lambda\overline{L}^\Lambda\right)\left(m^{J\Sigma}M_\Sigma - e^J_\Sigma L^\Sigma\right)+\\ &\quad +\omega^x_I\omega^x_J\left(m^{I\Lambda},e^I_\Lambda\right)\mathcal{S}\begin{pmatrix} m^{J\Sigma}\\ e^J_\Sigma\end{pmatrix}\end{aligned} \tag{2.16}$$

where the matrix $\mathcal{S}$ is a symplectic matrix given explicitly by:

$$\mathcal{S} = -\frac{1}{2}\begin{pmatrix} \Im_{\Lambda\Sigma}+\left(\Re\Im^{-1}\Re\right)_{\Lambda\Sigma} & -\left(\Re\Im^{-1}\right)_\Lambda{}^\Sigma \\ -\left(\Im^{-1}\Re\right)^\Lambda{}_\Sigma & \Im^{-1|\Lambda\Sigma}\end{pmatrix}. \tag{2.17}$$

$\Im_{\Lambda\Sigma}$ and $\Re_{\Lambda\Sigma}$ being the imaginary and real part of the period matrix $\mathcal{N}_{\Lambda\Sigma}$ of the special geometry. Furthermore the electric and magnetic charges must satisfy the constraint:

$$e^I_\Lambda m^{J\Lambda} - e^J_\Lambda m^{I\Lambda} = 0 \tag{2.18}$$

which follows from the Ward identity of supersymmetry.
As before the dependence of the scalar potential on the q^u's appears only through the SU(2)⊗SO(n_T) vector $\omega^x_I(q^u)$, while the dependence on z, $\overline{z}$ appears through the symplectic matrix of equation (2.16) and the covariantly holomorphic sections L^Λ, M_Λ of the special geometry.
¿From the point of view of dualization of $N=2$ standard theory the electric charges e^I_Λ can be associated to the gauging of the translational group of the axionic symmetries with generators T_Λ and subsequent dualization of the axions, while the magnetic charges $m^{\Lambda I}$ can be thought to be associated with the isometry algebra of the same translation group, with generators T^Λ, in the dual theory. Indeed if we would start in the standard $N=2$ supergravity from a purely magnetic theory with magnetic vector potentials A_Λ, then the Killing vectors associated to the translational group of the axions would naturally have an upper index Λ as the generators of the translation group of the dual theory.
In the case of Calabi–Yau compactification from Type IIB string theory the scalar potential (2.16) simplifies due to the important cancellation $\mathcal{M}_{IJ}-\omega^x_I\omega^x_J = 0$ [14]. Moreover [7], [14], [5] one finds that the tensors ω^x_I are given by [15]:

$$\omega^{(1)}_1 = 0; \quad \omega^{(2)}_1 = 0; \quad \omega^{(3)}_1 = e^\varphi$$

$$\omega_2^{(1)} = -e^{\varphi}\mathrm{Im}\tau; \quad \omega_2^{(2)} = 0; \quad \omega_2^{(3)} = e^{\varphi}\mathrm{Re}\tau. \tag{2.19}$$

where φ is the quaternionic scalar field defined as $\tilde{K}$ in reference [15]. Using the definition (2.19) one easily sees that the scalar potential for Calabi–Yau compactifications can be rewritten as:

$$\mathcal{V}_{CY} = -\frac{1}{2}e^{2\varphi}\left[\left(e_\Lambda - \overline{\mathcal{N}}_{\Lambda\Sigma}m^\Sigma\right)\left(\mathrm{Im}\mathcal{N}\right)^{-1|\Lambda\Gamma}\left(\overline{e}_\Gamma - \mathcal{N}_{\Gamma\Delta}\overline{m}^\Delta\right)\right] \tag{2.20}$$

where

$$e_\Lambda \equiv e^1_\Lambda + \tau e^2_\Lambda; \qquad m^\Lambda \equiv m^{1\Lambda} + \tau m^{2\Lambda}, \tag{2.21}$$

τ being the ten dimensional complex dilaton. This is exactly the potential appearing in the quoted references, where the constraint (2.18) reduces to the "tadpole cancellation condition"

$$e^1_\Lambda m^{2\Lambda} - e^2_\Lambda m^{1\Lambda} = 0 \tag{2.22}$$

We note that the constraint (2.18) (generalized tadpole cancellation condition) is now a consequence of the supersymmetry Ward identity (3.41) that one uses to compute the scalar potential [9].
If one thinks of this theory as coming from dualization of the axionic coordinates q^I of the quaternionic manifold of the standard $N = 2$ supergravity, the electric charges would appear as due to the gauging of the translational group of isometries $q^I \to q^I + c^I$. Indeed the quaternionic prepotential appearing in the gauge shifts of the fermions would be given in this case by the formula:

$$\mathcal{P}^x_\Lambda = \omega^x_I e^I_\Lambda \tag{2.23}$$

e^I_Λ being the constant Killing vectors of the translations [11].
Note that the procedure adopted in reference [1] does not seem suitable in order to find the Abelian electric charges e^I_Λ. In fact, it is evident that, even though formally they give rise to the same kind of structures [1], the Abelian electric charges e^I_Λ which pair together with the masses (or magnetic charges) $m^{I\Lambda}$ to reconstruct symplectic invariant structures [9], [7], [5], can not descend from the gauging of Abelian isometries of $\mathcal{M}_T$ since after dualization such axionic symmetries are not present on the residual σ–model on $\mathcal{M}_T$. One can understand it by considering the relation with standard

$N=2$ gauged supergravity [9] or just observing that if the Abelian electric charges must combine into a symplectic vector with the mass parameters

$$\mathcal{K}^I = \begin{pmatrix} m^{I\Lambda} \\ e^I_\Lambda \end{pmatrix} \tag{2.24}$$

they must carry the same index I of the tensors, that is of the axions which have been dualized. Even if we suppose that not all the axions have been dualized and thus there are some translational isometries left on the reduced manifold, the corresponding Killing vectors would not carry the index $I = 1, \ldots, n_T$.

3 The Lagrangian

In this section we present the Lagrangian and the supersymmetry transformation laws under which it is invariant. The method we used to derive the Lagrangian is the so called geometrical or rheonomic approach which is greatly facilitated when the solution of Bianchi identities in superspace is already known. The superspace geometrical Lagrangian and the solution of the Bianchi identities in superspace are shortly discussed in Appendix B. We limit ourselves in the main text to give the Lagrangian and the supersymmetry transformation laws up to 4–fermions and 3–fermions respectively, while the completion of the formulae containing quadrilinear and trilinear fermions are given in Appendix A. Our result is the following[2]

$$S = \int \mathrm{d}^4x \sqrt{-g}\, \mathcal{L} \tag{3.1}$$

$$\mathcal{L} = \mathcal{L}_{\mathrm{bos}} + \mathcal{L}^{\mathrm{kin}}_{\mathrm{ferm}} + \mathcal{L}_{\mathrm{Pauli}} + \mathcal{L}_{\mathrm{shifts}} + \mathcal{L}_{\mathrm{4f}} \tag{3.2}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{bos}} \;=\;& -\frac{1}{2}\mathcal{R} + g_{i\bar{\jmath}}\partial^\mu z^i \partial_\mu \overline{z}^{\bar{\jmath}} + \mathrm{i}\left(\overline{\mathcal{N}}_{\Lambda\Sigma}\hat{\mathcal{F}}^{-\Lambda}_{\mu\nu}\hat{\mathcal{F}}^{-\Lambda\mu\nu} - \mathcal{N}_{\Lambda\Sigma}\hat{\mathcal{F}}^{+\Lambda}_{\mu\nu}\hat{\mathcal{F}}^{+\Lambda\mu\nu}\right)\\ & +6\mathcal{M}^{IJ}\mathcal{H}_{I\mu\nu\rho}\mathcal{H}_J{}^{\mu\nu\rho} + g_{uv}\partial^\mu q^u \partial_\mu q^v + 2h^\mu_I A^I_u \nabla_\mu q^u +\\ & -2e^I_\Lambda\left(\hat{\mathcal{F}}^\Lambda_{\mu\nu} - m^{J\Lambda}B_{J\mu\nu}\right)B_{I\rho\sigma}\frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} - \mathcal{V}(z,\overline{z},q^u)\end{aligned} \tag{3.3}$$

[2]For the notations of $N=2$ theory and special geometry we refer the reader to the standard papers [11], [10]

$$
\begin{aligned}
\mathcal{L}_{\mathrm{ferm}}^{\mathrm{kin}} &= \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}}\left(\overline{\psi}^{A}_{\mu}\gamma_{\nu}\rho_{A|\lambda\sigma}-\overline{\psi}_{A|\mu}\gamma_{\nu}\rho^{A}_{\lambda\sigma}\right)-\frac{\mathrm{i}}{2}g_{i\bar{\jmath}}\left(\overline{\lambda}^{iA}\gamma^{\mu}\nabla_{\mu}\lambda^{\bar{\jmath}}_{A}+\right.\\
&\quad\left.+\overline{\lambda}^{\bar{\jmath}}_{A}\gamma^{\mu}\nabla_{\mu}\lambda^{iA}\right)-\mathrm{i}\left(\overline{\zeta}^{\alpha}\gamma^{\mu}\nabla_{\mu}\zeta_{\alpha}+\overline{\zeta}_{\alpha}\gamma^{\mu}\nabla_{\mu}\zeta^{\alpha}\right) \qquad (3.4)\\
\mathcal{L}_{\mathrm{Pauli}} &= -g_{i\bar{\jmath}}\left(\partial^{\mu}z^{i}\overline{\psi}^{A}_{\mu}\lambda^{\bar{\jmath}}_{A}+\partial_{\mu}z^{i}\overline{\psi}^{A}_{\nu}\gamma^{\mu\nu}\lambda^{\bar{\jmath}}_{A}+\mathrm{c.c.}\right)\\
&\quad-2\left(P_{u\,A\alpha}\partial^{\mu}q^{u}\overline{\psi}^{A}_{\mu}\zeta^{\alpha}+P_{u\,A\alpha}\partial_{\mu}q^{u}\overline{\psi}^{A}_{\nu}\gamma^{\mu\nu}\zeta^{\alpha}+\mathrm{c.c.}\right)\\
&\quad+6\mathrm{i}\mathcal{M}^{IJ}\mathcal{H}_{I}{}^{\mu\nu\rho}\left[\mathcal{U}_{J}{}^{A\alpha}\overline{\psi}_{A|\mu}\gamma_{\nu\rho}\zeta_{\alpha}-\mathcal{U}_{J\,A\alpha}\overline{\psi}^{A}_{\mu}\gamma_{\nu\rho}\zeta^{\alpha}\right]+\\
&\quad-24\mathrm{i}\mathcal{M}^{IJ}h_{I}{}^{\mu}\Delta_{J}{}^{\alpha}{}_{\beta}\overline{\zeta}_{\alpha}\gamma_{\mu}\zeta_{\beta}+24\mathrm{i}A^{I}_{u}\Delta_{I}{}^{\alpha}{}_{\beta}\overline{\zeta}_{\alpha}\gamma^{\mu}\zeta_{\beta}\nabla_{\mu}q^{u}+\\
&\quad+\left\{\hat{\mathcal{F}}^{-\Lambda}_{\mu\nu}\left(\mathrm{Im}\mathcal{N}\right)_{\Lambda\Sigma}\left[4L^{\Sigma}\left(\overline{\psi}^{A|\mu}\psi^{B|\nu}\right)^{-}\epsilon_{AB}+4\mathrm{i}f^{\Sigma}_{\bar{\imath}}\left(\overline{\lambda}^{\bar{\imath}}_{A}\gamma^{\mu}\psi^{\nu}_{B}\right)^{-}\epsilon^{AB}\right.\right.\\
&\quad\left.\left.+\frac{1}{2}\nabla_{i}f^{\Sigma}_{j}\overline{\lambda}^{iA}\gamma^{\mu\nu}\lambda^{jB}\epsilon_{AB}-L^{\Sigma}\overline{\zeta}_{\alpha}\gamma^{\mu\nu}\zeta_{\beta}\mathbb{C}^{\alpha\beta}\right]+\mathrm{c.c.}\right\}+\\
&\quad+2\mathcal{M}^{IJ}h^{\mu}_{I}\left(\mathcal{U}_{J}{}^{A\alpha}\overline{\psi}_{A|\mu}\zeta_{\alpha}+\mathcal{U}_{J\,A\alpha}\overline{\psi}^{A}_{\mu}\zeta^{\alpha}\right)+\\
&\quad+2\mathrm{i}\mathcal{M}^{IJ}h^{\mu}_{I}\Delta_{J\,\alpha}{}^{\beta}\zeta_{\beta}\gamma_{\mu}\zeta^{\alpha} \qquad (3.5)\\
\mathcal{L}_{\mathrm{shifts}} &= \left(2\overline{S}^{AB}\overline{\psi}^{\mu}_{A}\gamma_{\mu\nu}\psi^{\nu}_{B}+\mathrm{i}g_{i\bar{\jmath}}W^{iAB}\overline{\lambda}^{\bar{\jmath}}_{A}\gamma_{\mu}\psi^{\mu}_{B}+2\mathrm{i}N^{A}_{\alpha}\overline{\zeta}^{\alpha}\gamma_{\mu}\psi^{\mu}_{A}+\right.\\
&\quad+\frac{1}{2}\nabla_{u}N^{A}_{\alpha}P^{u}{}_{A\beta}\overline{\zeta}^{\alpha}\zeta^{\beta}+2\nabla_{\bar{\imath}}N^{A}_{\alpha}\overline{\zeta}^{\alpha}\lambda^{\bar{\imath}}_{A}+\\
&\quad\left.+\frac{1}{3}g_{i\bar{\jmath}}\nabla_{k}W^{\bar{\jmath}}_{AB}\overline{\lambda}^{iA}\lambda^{kB}\right)+\mathrm{c.c.}+\mathcal{L}^{\mathrm{non\,inv}}_{4\mathrm{f}}+\mathcal{L}^{\mathrm{inv}}_{4\mathrm{f}} \qquad (3.6)
\end{aligned}
$$

where the scalar potential

$$
\begin{aligned}
\mathcal{V} &= 4\left(\mathcal{M}_{IJ}-\omega^{x}_{I}\omega^{x}_{J}\right)\left(m^{I\Lambda}\overline{M}_{\Lambda}-e^{I}_{\Lambda}\overline{L}^{\Lambda}\right)\left(m^{J\Sigma}M_{\Sigma}-e^{J}_{\Sigma}L^{\Sigma}\right)+\\
&\quad+\omega^{x}_{I}\omega^{x}_{J}\left(m^{I\Lambda},e^{I}_{\Lambda}\right)\mathcal{S}\begin{pmatrix}m^{J\Sigma}\\ e^{J}_{\Sigma}\end{pmatrix} \qquad (3.7)
\end{aligned}
$$

coincides with the expression given in equation (2.16). We have rewritten it in the Lagrangian for completeness. Notations are as follows (for what is concerned the special geometry quantities we use the standard notations of reference [11], in particular i,j, $\bar{\imath},\bar{\jmath}$ are curved indices on the special Kähler manifold): covariant derivatives on the fermions are defined in terms of the 1–form SU(2)–connection ω^{AB}, the Kähler U(1) connection, Q, the Sp($2n_H$)

connection $\Delta^{\alpha\beta}$ and Christoffel connection $\Gamma^i{}_j$ defined on $\mathcal{M}_{SK} \otimes \mathcal{M}_T$ (covariant derivatives are defined in Appendix B). Note that f_j^Σ being part of the symplectic section of special geometry has a derivative covariant with respect to the Kähler connection. Furthermore $P_{uA\alpha}$ is a "rectangular vielbein" [1] related to the metric g_{uv} of the σ–model by the relation $P_u{}^{A\alpha} P_{vA\alpha} = g_{uv}$, and $P^{uA\alpha} = g^{uv} P_v{}^{A\alpha}$.
Besides the field–strengths of the tensors, all the I, J indexed quantities, namely $A_u^I, \mathcal{U}_I{}^{A\alpha}$, $\omega_{IA}{}^B$, $\Delta_{I\alpha}{}^\beta$ are vectors on an SO(n_T) bundle defined on the σ–model, whose metric is $\mathcal{M}_{IJ}$. We note that, if one thinks of this Lagrangian as coming from the $N = 2$ standard supergravity [11], all these I–indexed quantities can be interpreted as the remnants of the original quaternionic metric $h_{\hat{u}\hat{v}}$ (M_{IJ}, A_u^I), vielbein $\mathcal{U}_{\hat{u}}^{A\alpha}$ ($\mathcal{U}_I{}^{A\alpha}$), SU(2) and Sp$(2n)$ 1–form connections ω^{AB} and $\Delta^{\alpha\beta}$ after dualization of the q^I coordinates ($q^{\hat{u}} = (q^u, q^I)$) of the quaternionic manifold. Moreover we have defined:

$$
\begin{aligned}
\mathcal{F}^\Lambda_{\mu\nu} &= \partial_{[\mu} A^\Lambda_{\nu]} && (3.8)\\
\mathcal{F}^{\pm\Lambda}_{\mu\nu} &= \frac{1}{2}\left(\mathcal{F}^\Lambda_{\mu\nu} \pm \frac{\mathrm{i}}{2}\epsilon_{\mu\nu\rho\sigma}\mathcal{F}^{\Lambda\rho\sigma}\right) && (3.9)\\
\mathcal{H}_{I\mu\nu\rho} &= \partial_{[\mu} B_{I\,\nu\rho]} && (3.10)\\
h_{I\mu} &= \epsilon_{\mu\nu\rho\sigma}\mathcal{H}_I{}^{\nu\rho\sigma}. && (3.11)
\end{aligned}
$$

The Lagrangian (3.2) is invariant under the following supersymmetry transformations:

$$
\begin{aligned}
\delta\psi_{A|\mu} &= \mathcal{D}_\mu \epsilon_A - h^I_\mu \omega_{I\,A}{}^B \varepsilon_B + \\
&\quad + \left[\mathrm{i} S_{AB}\eta_{\mu\nu} + e_{AB} T^-_{\mu\nu}\right]\gamma^\nu \epsilon^B + 3\,\text{fermions} && (3.12)\\
\delta\lambda^{iA} &= \mathrm{i}\left(\nabla_\mu z^i\right)\gamma^\mu\epsilon^A + G^{-i}_{\mu\nu}\gamma^{\mu\nu}\epsilon_B\epsilon^{AB} + W^{iAB}\epsilon_B + 3\,\text{fermions} && (3.13)\\
\delta\zeta_\alpha &= \mathrm{i} P_{uA\alpha}\partial_\mu q^u \gamma^\mu \varepsilon^A - \mathrm{i} h^I_\mu \mathcal{U}_{IA\alpha}\gamma^\mu\varepsilon^A + N^A_\alpha \varepsilon_A + 3\,\text{fermions} && (3.14)\\
\delta V^a_\mu &= -\mathrm{i}\overline{\psi}_{A\mu}\gamma^a\epsilon^A - \mathrm{i}\overline{\psi}^A_\mu \gamma^a \epsilon_A && (3.15)\\
\delta A^\Lambda_\mu &= 2L^\Lambda \overline{\psi}^A_\mu \epsilon^B \epsilon_{AB} + 2\overline{L}^\Lambda \overline{\psi}_{A\mu}\epsilon_B\epsilon^{AB} + \\
&\quad + \left(\mathrm{i} f^\Lambda_i \overline{\lambda}^{iA}\gamma_\mu\epsilon^B\epsilon_{AB} + \mathrm{i}\overline{f}^\Lambda_{\bar{\imath}}\overline{\lambda}^{\bar{\imath}}_A \gamma_\mu \epsilon_B \epsilon^{AB}\right) && (3.16)\\
\delta B_{I\mu\nu} &= -\frac{\mathrm{i}}{2}\left(\overline{\varepsilon}_A \gamma_{\mu\nu}\zeta_\alpha \mathcal{U}_I{}^{A\alpha} - \overline{\varepsilon}^A \gamma_{\mu\nu}\zeta^\alpha \mathcal{U}_{IA\alpha}\right) +
\end{aligned}
$$

$$
\begin{aligned}
& -\omega_{IC}{}^{A}\left(\overline{\varepsilon}_{A}\gamma_{[\mu}\psi^{C}_{\nu]}+\overline{\psi}_{[\mu A}\gamma_{\nu]}\varepsilon^{C}\right) && (3.17)\\
\delta z^{i} &= \overline{\lambda}^{iA}\epsilon_{A} && (3.18)\\
\delta\overline{z}^{\overline{\imath}} &= \overline{\lambda}^{\overline{\imath}}_{A}\epsilon^{A} && (3.19)\\
\delta q^{u} &= P^{u}{}_{A\alpha}\left(\overline{\zeta}^{\alpha}\epsilon^{A}+\mathbb{C}^{\alpha\beta}\epsilon^{AB}\overline{\zeta}_{\beta}\epsilon_{B}\right) && (3.20)
\end{aligned}
$$

where the dressed field–strengths appearing in the transformation laws of the gravitino and gaugino fields are defined as:

$$
\begin{aligned}
T^{-}_{\mu\nu} &= 2\mathrm{i}\,(\mathrm{Im}\mathcal{N})_{\Lambda\Sigma}\,L^{\Sigma}\hat{\mathcal{F}}^{\Lambda-}_{\mu\nu} + \text{ bilinear fermions} && (3.21)\\
G^{i-}_{\mu\nu} &= -g^{i\overline{\jmath}}\,\overline{f}^{\Gamma}_{\overline{\jmath}}\,(\mathrm{Im}\mathcal{N})_{\Gamma\Lambda}\,\hat{\mathcal{F}}^{\Lambda-}_{\mu\nu} + \text{ bilinear fermions} && (3.22)
\end{aligned}
$$

where the bilinear fermions are the same as in standard $N=2$ supergravity and are given explicitly in the Appendix 3.2. We have written the transformation laws only for the chiral spinor fields ($\psi_{A\mu}$, λ^{iA}, ζ_{α}); the transformation laws for the anti–chiral fields (ψ^{A}_{μ}, $\lambda^{\overline{\imath}}_{A}$, ζ^{α}) are immediately obtained from the chiral ones.
Finally, the shift matrices S_{AB}, $W^{i\,AB}$ and N^{A}_{α} are given by equations (2.13, 2.14,2.15). We also note that the given Lagrangian is invariant under the gauge transformation

$$
\begin{aligned}
\delta B_{I} &= \mathrm{d}\Lambda_{I}\\
\delta A^{\Lambda} &= -2m^{I\Lambda}\Lambda_{I}
\end{aligned}
$$

where Λ_{I} is a 1–form, if and only if the following generalized tadpole cancellation condition is satisfied

$$
e^{I}_{\Lambda}m^{J\Lambda}-e^{J}_{\Lambda}m^{I\Lambda}=0. \tag{3.23}
$$

Indeed $\hat{\mathcal{F}}^{\Lambda}$ is invariant by itself while the topological term is invariant only if equation (2.18) is satisfied. As observed in the previous section, the same consistency condition also appears from the supersymmetry Ward identity from which one can compute the scalar potential.
As far as the symplectic invariance is concerned [16], nothing is changed with respect to the standard $N=2$ supergravity. Indeed if we consider the bosonic part of the Lagrangian for the vector and tensor fields, namely:

$$\begin{aligned}
\mathcal{L}^{\rm bos}_{\rm vect} + \mathcal{L}^{\rm bos}_{\rm top} &= {\rm i}\left(\overline{\mathcal{N}}_{\Lambda\Sigma}\hat{\mathcal{F}}^{\Lambda -}_{\mu\nu}\hat{\mathcal{F}}^{\Sigma - \mu\nu} - \mathcal{N}_{\Lambda\Sigma}\hat{\mathcal{F}}^{\Lambda +}_{\mu\nu}\hat{\mathcal{F}}^{\Sigma + \mu\nu}\right)\\
&-4{\rm i}e^I_\Lambda\left(\hat{\mathcal{F}}^{\Lambda -}_{\mu\nu}B_I^{-\mu\nu} - \hat{\mathcal{F}}^{\Lambda +}_{\mu\nu}B_I^{+\mu\nu}\right) +\\
&+4{\rm i}m^{J\Lambda}e^I_\Lambda B_J^{-\mu\nu}B^-_{I\mu\nu} - 4{\rm i}m^{J\Lambda}e^I_\Lambda B_J^{+\mu\nu}B^+_{I\mu\nu}
\end{aligned} \tag{3.24}$$

and define

$$\mathcal{G}^{(\rm bos)\mp}_{\Lambda\mu\nu} = \mp\frac{\rm i}{2}\frac{\delta\mathcal{L}^{(\rm bos)}}{\delta\hat{\mathcal{F}}^{\Lambda\mp\mu\nu}} = \overline{\mathcal{N}}_{\Lambda\Sigma}\hat{\mathcal{F}}^{\Sigma\mp}_{\mu\nu} - 2e^I_\Lambda B^\mp_{I\mu\nu} \tag{3.25}$$

then equation (3.24) can be rewritten as

$$\begin{aligned}
&{\rm i}\left(\hat{\mathcal{F}}^{\Lambda -}_{\mu\nu}\mathcal{G}^{(\rm bos)\,-\mu\nu}_\Lambda - 2e^I_\Lambda B^-_{I\mu\nu}\hat{\mathcal{F}}^{\Lambda - \mu\nu} + 4e^I_\Lambda m^{J\Lambda}B^-_{I\mu\nu}B_J^{-\mu\nu}\right) + {\rm c.c.} =\\
&\quad = {\rm i}\left[\mathcal{F}^{\Lambda -}\mathcal{G}^{(\rm bos)\,-}_\Lambda + 2B^-_I\left(m^{I\Lambda}\mathcal{G}^-_\Lambda - e^I_\Lambda\mathcal{F}^{\Lambda -}\right)\right] + {\rm c.c.}
\end{aligned} \tag{3.26}$$

The first term in (3.26) is the usual vector kinetic term of the standard $N = 2$ theory while the additional term is a symplectic invariant. Indeed one can easily verify that $\mathcal{G}^{(\rm bos)\,-}_\Lambda$, as defined in (3.25), transforms as the lower component of the symplectic vector ($\mathcal{F}^{\Lambda -}$, $\mathcal{G}^{(\rm bos)\,-}_\Lambda$), provided $\mathcal{N}_{\Lambda\Sigma}$ transforms under the symplectic transformation (2.9) as usual, namely:

$$\mathcal{N}'_{\Lambda\Sigma} = (C + D\mathcal{N})(A + B\mathcal{N})^{-1} \tag{3.27}$$

On the other hand the vector ($m^{I\Lambda}$, e^I_Λ) was already supposed to transform in the same way as the vector (L^Λ, M_Λ) as does the symplectic vector ($\mathcal{F}^{\Lambda -}$, $\mathcal{G}^{\Lambda -}$) of the special geometry.
If we also take into account the fermionic sector of the Lagrangian then $\mathcal{G}^{(\rm bos)}$ is completed with bilinear fermions:

$$\begin{aligned}
\mathcal{G}^-_{\Lambda\mu\nu} &= \mathcal{G}^{(\rm bos)-}_{\Lambda\mu\nu} - \frac{\rm i}{2}({\rm Im}\mathcal{N})_{\Lambda\Sigma}\mathcal{D}^{\Sigma -}_{\mu\nu}\\
\mathcal{G}^+_{\Lambda\mu\nu} &= \left(\mathcal{G}^-_{\Lambda\mu\nu}\right)^*
\end{aligned} \tag{3.28}$$

where $\mathcal{D}^{\Lambda -}_{\mu\nu}$ is the coefficient of $\mathcal{F}^{\Sigma -}_{\mu\nu}{\rm Im}\mathcal{N}_{\Lambda\Sigma}$ in $\mathcal{L}_{\rm Pauli}$. Following the argument of reference [17], the full Lagrangian is on–shell symplectic invariant (or invariant up to the $\mathcal{F}\mathcal{G}$ term) by adding non–symplectic invariant 4–fermion

terms $\mathcal{L}_{4f}^{\mathrm{non\,inv}}$. This non–invariant 4–fermion terms are exactly the same as in the standard $N = 2$ theory because so is the term $-\frac{\mathrm{i}}{2}(\mathrm{Im}\mathcal{N})_{\Lambda\Sigma}\mathcal{D}^{\Sigma-}_{\mu\nu}$ and their explicit form is given in the Appendix A. $\mathcal{L}_{4f}^{\mathrm{inv}}$ is instead fixed only by supersymmetry. Its explicit form again coincides with that given in the standard theory except for some additional terms as explained in Appendix A.
We also note that the equations of motion and Bianchi identities for the vectors have exactly the same form as in the standard theory, namely:

$$\partial_\mu \mathrm{Im}\mathcal{F}^{\Lambda-\,\mu\nu} = 0 \tag{3.29}$$

$$\partial_\mu \mathrm{Im}\mathcal{G}^{-\,\mu\nu}_{\Lambda} = 0 \tag{3.30}$$

where $\mathcal{G}^{-}_{\Lambda\mu\nu}$ differs from the analogous one of the standard $N = 2$ theory by the term linear in $B_{I\mu\nu}$ appearing in the equation (3.25), and they are symplectic covariant.
Finally we note that the generalized tadpole cancellation condition (2.18) implies that the symplectic vectors $m^{I\Lambda}, e^I_\Lambda$ are all parallel. Therefore by a symplectic rotation we can choose the gauge where all the magnetic charges $m^{I\Lambda}$ are zero. In this case we are in a strictly perturbative regime. In an analogous way we could also rotate the charge vector in such a way that all the electric charges e^I_Λ are zero and we may think of this regime as a perturbative regime for a purely magnetic theory.

3.1 Adding a semisimple gauging

It is now immediate to generalize the theory given in the previous section by gauging the group of the isometries of the σ–model parametrized by the scalars q^u of the scalar tensor multiplet. Let us call $\mathcal{G}$ the group of isometries that can be gauged on such a manifold. The gauging due to $\mathcal{G}$ was given in a general form in reference [1] where however there was no consideration of the Fayet–Iliopoulos terms giving rise to the electric charges. Such a gauging breaks of course the symplectic invariance of the equations of motion since it involves only the gauging associated to electric vector potentials. In order to distinguish between the Fayet–Iliopoulos terms and the new terms associated to the gauging we split the index Λ into $(\Lambda = \hat{\Lambda}, \check{\Lambda})$, where $\hat{\Lambda} = 1, \ldots, n_T$ is used for the electric and magnetic charges and the related Special Geometry sections associated to the deformations coming from Fayet–Iliopoulos terms, while we denote by $\check{\Lambda}$, the index running on the adjoint representation of

the group $\mathcal{G}$, associated to the electric vector potentials $A_\mu^{\check\Lambda}$ that are gauged. Referring to reference [1], we know that the fermionic shifts acquire a further term so that the total shifts are now given by

$$
\begin{aligned}
S_{AB} &= \frac{\mathrm{i}}{2}\,\sigma^x_{AB}\,g\,\mathcal{P}^x_{\check\Lambda}L^{\check\Lambda} + \frac{\mathrm{i}}{2}\,\sigma^x_{AB}\,\omega^x_I\left(e^I_{\hat\Lambda}L^{\hat\Lambda} - m^{I\hat\Lambda}M_{\hat\Lambda}\right) && (3.31)\\
N^\alpha_A &= -2\,g\,\left(P_{uA}{}^\alpha + A^I_u\mathcal{U}_{IA}{}^\alpha\right)k^u_{\check\Lambda}L^{\check\Lambda} - 2\,\mathcal{U}_{IA}{}^\alpha\left(e^I_{\hat\Lambda}L^{\hat\Lambda} - m^{I\hat\Lambda}M_{\hat\Lambda}\right) && (3.32)\\
W^{iAB} &= -\mathrm{i}\,g\,\epsilon^{AB}g^{i\bar\jmath}P_{\check\Lambda}f^{\check\Lambda}_{\bar\jmath} + \mathrm{i}\,g^{i\bar\jmath}\sigma^{x\,AB}g\mathcal{P}^x_{\check\Lambda}\overline{f}^{\check\Lambda}_{\bar\jmath} + \\
&\quad + \mathrm{i}\,g^{i\bar\jmath}\sigma^{x\,AB}\omega^x_I\left(e^I_{\hat\Lambda}\overline{f}^{\hat\Lambda}_{\bar\jmath} - m^{I\hat\Lambda}h_{\hat\Lambda\bar\jmath}\right) && (3.33)
\end{aligned}
$$

which in fact reduce to those given in reference [1] if we set $e^I_{\hat\Lambda} = 0$. Here $P_{\check\Lambda}$ is the prepotential of the special geometry, $\mathcal{P}^x_{\hat\Lambda}$ is a new quantity which from the point of view of dualization of standard $N = 2$ supergravity, can be thought as the quaternionic prepotential restricted to $\mathcal{M}_T$ and $k^u_{\check\Lambda}$ is the Killing vector of the gauged isometries. As we pointed out before, from the point of view of dualizing the standard $N = 2$ theory with quaternions the terms in $e^I_{\hat\Lambda}$ appearing in the shifts can be thought of as coming from a translational gauging of the axionic symmetries on the quaternionic manifold before dualization so that, from this point of view, the gauge group which is gauged is $\mathcal{T}\otimes\mathcal{G}$ where $\mathcal{T}$ is an Abelian translation group gauging the axionic symmetries.

The Lagrangian for the gauged theory can be written formally in exactly the same way as the Lagrangian (3.2) provided we make the following modification (for more details see [11]): the covariant derivatives on the fermions are now defined in terms of the gauged connections $\hat\omega^{AB}$, $\hat\Delta^{\alpha\beta}$, $\Gamma^i{}_j$ defined in reference [11] while the ordinary derivatives acting on the scalars z^i and q^u are now replaced by gauge derivatives:

$$
\begin{aligned}
\partial_\mu q^u &\rightarrow \nabla_\mu q^u \equiv \partial_\mu q^u + k^u_{\hat\Lambda}A^{\hat\Lambda}_\mu && (3.34)\\
\partial_\mu z^i &\rightarrow \nabla_\mu z^i \equiv \partial_\mu z^i + k^i_{\hat\Lambda}A^{\hat\Lambda}_\mu && (3.35)\\
\partial_\mu \overline{z}^{\bar\imath} &\rightarrow \nabla_\mu \overline{z}^{\bar\imath} \equiv \partial_\mu \overline{z}^{\bar\imath} + k^{\bar\imath}_{\hat\Lambda}A^{\hat\Lambda}_\mu && (3.36)
\end{aligned}
$$

Furthermore the scalar potential takes additional terms which are discussed in the next section. In order not to have too messy formula we do not perform

the splitting $\Lambda \rightarrow \hat{\Lambda}, \check{\Lambda}$ on such generalized Lagrangian. However one must pay attention to the fact that saturation of indices Λ, Σ in the Lagrangian have to be done according to this splitting; in particular saturation of the Λ indices with electric and magnetic charges run only on $\hat{\Lambda}$ and furthermore we have of course

$$\begin{aligned} \hat{F}^{\check{\Lambda}} &= \mathrm{d}A^{\check{\Lambda}} + \frac{1}{2} f_{\check{\Sigma}\check{\Pi}}{}^{\check{\Lambda}} A^{\check{\Sigma}} A^{\check{\Pi}} \qquad (3.37) \\ \hat{F}^{\hat{\Lambda}} &= \mathrm{d}A^{\hat{\Lambda}} + m^{I\hat{\Lambda}} B_I. \qquad (3.38) \end{aligned}$$

However the splitting $\Lambda \rightarrow \hat{\Lambda}, \check{\Lambda}$ will be performed explicitly in the next section for the discussion of the scalar potential in the presence of gauging.

3.2 The scalar potential for the gauged theory

The explicit form of the scalar potential is:

$$\begin{aligned} \mathcal{V} &= g_{i\bar{\jmath}} k^i_{\check{\Lambda}} k^{\bar{\jmath}}_{\check{\Sigma}} \overline{L}^{\check{\Lambda}} L^{\check{\Sigma}} + 4 \left(m^{I\hat{\Lambda}}, e^I_{\hat{\Lambda}}, k^u_{\check{\Lambda}} \right) \mathcal{Q} \begin{pmatrix} m^{J\hat{\Sigma}} \\ e^J_{\hat{\Sigma}} \\ k^v_{\check{\Sigma}} \end{pmatrix} + \\ &+ \left(m^{I\hat{\Lambda}} \omega^x_I, e^I_{\hat{\Lambda}} \omega^x_I, \mathcal{P}^x_{\check{\Lambda}} \right) (\mathcal{R} + \mathcal{S}) \begin{pmatrix} m^{J\hat{\Sigma}} \omega^x_J \\ e^J_{\hat{\Sigma}} \omega^x_J \\ \mathcal{P}^x_{\check{\Sigma}} \end{pmatrix} \qquad (3.39) \end{aligned}$$

where we have defined the following matrices

$$\begin{aligned} \mathcal{Q} &= \begin{pmatrix} \mathcal{M}_{IJ} \overline{M}_{\hat{\Lambda}} M_{\hat{\Sigma}} & -\mathcal{M}_{IJ} \overline{M}_{\hat{\Lambda}} L^{\hat{\Sigma}} & -A^K_v \mathcal{M}_{IK} \overline{M}_{\hat{\Lambda}} L^{\check{\Sigma}} \\ -\mathcal{M}_{IJ} \overline{L}^{\hat{\Lambda}} M_{\hat{\Sigma}} & \mathcal{M}_{IJ} \overline{L}^{\hat{\Lambda}} L^{\hat{\Sigma}} & A^K_v \mathcal{M}_{KI} \overline{L}^{\hat{\Lambda}} L^{\check{\Sigma}} \\ -A^K_u \mathcal{M}_{KJ} \overline{L}^{\check{\Lambda}} M_{\hat{\Sigma}} & A^K_u \mathcal{M}_{KJ} \overline{M}_{\hat{\Sigma}} L^{\check{\Lambda}} & \left(g_{uv} + A^I_u A^J_v \mathcal{M}_{IJ} \right) \overline{L}^{\check{\Lambda}} L^{\check{\Sigma}} \end{pmatrix} \\ \mathcal{R} &= -4 \begin{pmatrix} \overline{M}_{\hat{\Lambda}} M_{\hat{\Sigma}} & -\overline{M}_{\hat{\Lambda}} L^{\hat{\Sigma}} & -\overline{L}^{\check{\Sigma}} M_{\hat{\Lambda}} \\ -\overline{L}^{\hat{\Lambda}} M_{\hat{\Sigma}} & \overline{L}^{\hat{\Lambda}} L^{\hat{\Sigma}} & \overline{L}^{\hat{\Lambda}} L^{\check{\Sigma}} \\ -L^{\check{\Lambda}} \overline{M}_{\hat{\Sigma}} & \overline{L}^{\check{\Lambda}} L^{\hat{\Sigma}} & \overline{L}^{\check{\Lambda}} L^{\check{\Sigma}} \end{pmatrix} \\ \mathcal{S} &= -\frac{1}{2} \begin{pmatrix} \Im_{\hat{\Lambda}\hat{\Sigma}} + (\Re \Im^{-1} \Re)_{\hat{\Lambda}\hat{\Sigma}} & -\left(\Re \Im^{-1} \right)_{\hat{\Lambda}}{}^{\hat{\Sigma}} & -\left(\Im^{-1} \Re \right)_{\hat{\Lambda}}{}^{\check{\Sigma}} \\ -\left(\Im^{-1} \Re \right)^{\hat{\Lambda}}{}_{\hat{\Sigma}} & \Im^{-1|\hat{\Lambda}\hat{\Sigma}} & \Im^{-1|\hat{\Lambda}\check{\Sigma}} \\ -\left(\Re \Im^{-1} \right)^{\check{\Lambda}}{}_{\hat{\Sigma}} & \Im^{-1|\check{\Lambda}\hat{\Sigma}} & \Im^{-1|\check{\Lambda}\check{\Sigma}} \end{pmatrix}, \qquad (3.40) \end{aligned}$$

and where we have set as before $\Im_{\Lambda\Sigma} = \mathrm{Im}\mathcal{N}_{\Lambda\Sigma}$, $\Re_{\Lambda\Sigma} = \mathrm{Re}\mathcal{N}_{\Lambda\Sigma}$.
The derivation of the scalar potential has been found from the supersymmetry Ward identity:

$$\delta^A_B \mathcal{V} \,=\, -12\, \overline{S}^{CA}\, S_{CB} \,+\, g_{i\bar{\jmath}} W^{iCA}\, W^{\bar{\jmath}}_{CB} \,+\, 2 N^A_\alpha\, N^\alpha_B \tag{3.41}$$

This identity implies that all the terms proportional to a Pauli σ–matrix must cancel against each other. We find besides the constraint (2.18) the following new constraint:

$$f_{\check{\Lambda}\check{\Sigma}}{}^{\check{\Delta}}\, (\mathrm{Im}\mathcal{N})_{\check{\Delta}\hat{\Pi}}\, m^{I\hat{\Pi}} = f_{\check{\Lambda}\check{\Sigma}}{}^{\check{\Delta}} \tilde{m}^I_{\check{\Delta}} = 0\,, \tag{3.42}$$

where

$$\tilde{m}^I_{\check{\Delta}} \equiv (\mathrm{Im}\mathcal{N})_{\check{\Delta}\hat{\Pi}}\, m^{I\hat{\Pi}}\,. \tag{3.43}$$

This condition was already discussed in reference [1]. There it was noted that this equation means that $\tilde{m}^I_{\check{\Delta}}$ are the coordinates of a Lie algebra element of the gauge group commuting with all the generators, in other words a non trivial element of the center $\mathbb{Z}(\mathcal{G})$. From the point of view of dualization of the standard $N = 2$ supergravity with gauged translations this condition has a clear interpretation. Indeed since our theory can be thought of as coming from the gauging of a group $\mathcal{T} \otimes \mathcal{G}$ with $\mathcal{T}$ translational group of the axionic symmetries this condition is certainly satisfied since $m^{I\hat{\Lambda}}$ is in the (dual) Lie algebra of $\mathcal{T}$.
The expression for the potential looks quite complicated; however we may reduce the given potential to known cases by suitable erasing of some of the rows and columns of the matrices $\mathcal{Q}$, $\mathcal{R}$, $\mathcal{S}$.
First of all we note that if we delete the third row and the third column of the given matrices we obtain the form of the potential in the pure translational case with mass deformations given by equation (2.16) (with $\Lambda \to \hat{\Lambda}$). If we further delete the first row and the first column, which means that we do not implement mass deformations so that the tensor fields are massless, then the potential reduces to [14]

$$\mathcal{V} = -\frac{1}{2}\, (\mathrm{Im}\mathcal{N})^{-1|\hat{\Lambda}\hat{\Sigma}}\, e^I_{\hat{\Lambda}} e^J_{\hat{\Sigma}} \omega^x_I \omega^x_J + 4\, (\mathcal{M}_{IJ} - \omega^x_I \omega^x_J)\, e^I_{\hat{\Lambda}} e^J_{\hat{\Sigma}} \overline{L}^{\hat{\Lambda}} L^{\hat{\Sigma}}. \tag{3.44}$$

Another possibility is to delete the first two rows and the first two columns, which means that we have no translational gauging but only the gauging due to $\mathcal{G}$. In this case the potential takes the form

$$\begin{aligned}\mathcal{V}_{\mathcal{G}} &= g_{i\bar{\jmath}} k^i_{\check{\Lambda}} k^{\bar{\jmath}}_{\check{\Sigma}} \overline{L}^{\check{\Lambda}} L^{\check{\Sigma}} + 4\left(g_{uv} + A^I_u A^J_v \mathcal{M}_{IJ}\right) k^u_{\check{\Lambda}} k^v_{\check{\Sigma}} \overline{L}^{\check{\Lambda}} L^{\check{\Sigma}} + \\ &\quad - \left(\frac{1}{2}\left(\mathrm{Im}\mathcal{N}\right)^{-1|\check{\Lambda}\check{\Sigma}} + 4\overline{L}^{\check{\Lambda}} L^{\check{\Sigma}}\right) \mathcal{P}^x_{\check{\Lambda}} \mathcal{P}^x_{\check{\Sigma}}. \end{aligned} \tag{3.45}$$

Note that this potential has exactly the same form as in the gauged standard $N = 2$ supergravity coupled to vector multiplets and hypermultiplets. The only difference is that the prepotentials and the Killing vectors appearing in the last two terms are now restricted to the manifold $\mathcal{M}_T$ of the scalar tensor multiplets. We also note that $g_{uv} + A^I_u A^J_v \mathcal{M}_{IJ}$ is simply the original quaternionic metric restricted to the σ–model. In an analogous way we could erase the first row and the first column of the given matrices we obtain the potential with gauging of group $\mathcal{T} \otimes \mathcal{G}$ but without mass deformations and similarly for the other possibilities.

Acknowledgments

We would like to thank Mario Trigiante for enlightening discussions.
The work of R. D'Auria and L. Sommovigo has been supported by the European Community's Human Potential Program under contract HPRN-CT-2000-00131 Quantum Space–Time where R.D. and L.S. are associated to Torino University.

Appendix A: Completion of the Lagrangian and transformation laws

In this Appendix we give the explicit expression of the 4–fermion terms, $\mathcal{L}_{\mathrm{4f}}$ of the Lagrangian of section 3 and the complete supersymmetry transformation laws of the fermions including 3–fermion terms.
As observed in section 3, the completion of the 4–fermion terms, $\mathcal{L}_{\mathrm{4f}}$ can be split in two pieces namely $\mathcal{L}^{\mathrm{non\,inv}}_{\mathrm{4f}}$ and $\mathcal{L}^{\mathrm{inv}}_{\mathrm{4f}}$.
$\mathcal{L}^{\mathrm{non\,inv}}_{\mathrm{4f}}$ can be fixed by the knowledge of the Pauli terms, and as explained in

the text, it is exactly the same as $\mathcal{L}_{4f}^{\text{non inv}}$ of the standard $N=2$ supergravity [11]. We have:

$$
\begin{aligned}
\mathcal{L}_{4f}^{\text{non inv}} \;=\; & \Big\{ (\mathrm{Im}\mathcal{N})_{\Lambda\Sigma} \Big[2L^\Lambda L^\Sigma \left(\overline{\psi}^A_\mu \psi^B_\nu\right)^- \left(\overline{\psi}^C_\mu \psi^D_\nu\right)^- \epsilon_{AB}\epsilon_{CD} + \\
& -8\mathrm{i} L^\Lambda \overline{f}^\Sigma_{\bar\imath} \left(\overline{\psi}^A_\mu \psi^B_\nu\right)^- \left(\overline{\lambda}^{\bar\imath}_A \gamma^\nu \psi^\mu_B\right)^- + \\
& -2\overline{f}^\Lambda_{\bar\imath}\overline{f}^\Sigma_{\bar\jmath} \left(\overline{\lambda}^{\bar\imath}_A \gamma^\nu \psi^\mu_B\right)^- \left(\overline{\lambda}^{\bar\jmath}_C \gamma_\nu \psi_{D|\mu}\right)^- \epsilon^{AB}\epsilon^{CD} + \\
& +\frac{\mathrm{i}}{2} L^\Lambda \overline{f}^\Sigma_{\bar l}\, g^{k\bar l} C_{ijk} \left(\overline{\psi}^A_\mu \psi^B_\nu\right)^- \overline{\lambda}^{iC} \gamma^{\mu\nu} \lambda^{jD} \epsilon_{AB}\epsilon_{CD} + \\
& +\overline{f}^\Lambda_{\bar m}\overline{f}^\Sigma_{\bar l}\, g^{k\bar l} C_{ijk} \left(\overline{\lambda}^{\bar m}_A \gamma_\nu \psi_{B|\mu}\right)^- \overline{\lambda}^{iA} \gamma^{\mu\nu} \lambda^{jB} + \\
& -L^\Lambda L^\Sigma \left(\overline{\psi}^A_\mu \psi^B_\nu\right)^- \overline{\zeta}_\alpha \gamma^{\mu\nu} \zeta_\beta\, \epsilon_{AB}\, \mathbb{C}^{\alpha\beta} + \\
& +\mathrm{i} L^\Lambda \overline{f}^\Sigma_{\bar\imath} \left(\overline{\lambda}^{\bar\imath}_A \gamma^\nu \psi^\mu_B\right)^- \overline{\zeta}_\alpha \gamma_{\mu\nu} \zeta_\beta\, \epsilon^{AB}\, \mathbb{C}^{\alpha\beta} + \\
& -\frac{1}{32} C_{ijk}\, C_{lmn}\, g^{k\bar r}\, g^{n\bar s}\, \overline{f}^\Lambda_{\bar r}\, \overline{f}^\Sigma_{\bar s}\, \overline{\lambda}^{iA} \gamma_{\mu\nu} \lambda^{jB}\, \overline{\lambda}^{kC} \gamma^{\mu\nu} \lambda^{lD} \epsilon_{AB}\, \epsilon_{CD} + \\
& -\frac{1}{8} L^\Lambda \nabla_i f^\Sigma_j\, \overline{\zeta}_\alpha \gamma_{\mu\nu} \zeta_\beta\, \overline{\lambda}^{iA} \gamma^{\mu\nu} \lambda^{jB} \epsilon_{AB} \mathbb{C}^{\alpha\beta} + \\
& +\frac{1}{8} L^\Lambda L^\Sigma\, \overline{\zeta}_\alpha \gamma_{\mu\nu} \zeta_\beta\, \overline{\zeta}_\gamma \gamma^{\mu\nu} \zeta_\delta\, \mathbb{C}^{\alpha\beta}\, \mathbb{C}^{\gamma\delta} \Big] + \text{c.c.} \Big\}
\end{aligned}
\tag{A.1}
$$

Of course the same terms can be obtained by supersymmetry. The residual 4–fermion terms $\mathcal{L}_{4f}^{\text{noninv}}$ are instead only fixed by supersymmetry. It is clear that this terms are exactly the same as in the standard $N=2$ theory provided one makes the following modifications:

1. First of all we have terms quadrilinear in the ζ_α coming from the fact that dualization on space–time of the axionic coordinates q^I gives, together with the 3–form $\mathcal{H}_{I\mu\nu\rho}$ also the bilinear $2\mathrm{i}\,\Delta_I{}^\alpha{}_\beta\,\zeta_\alpha\gamma^\sigma\zeta^b\epsilon_{\sigma\mu\nu\rho}$.

2. One has also to reduce the symplectic curvature appearing in the following 4–fermion term of $N=2$ standard supergravity

$$
\frac{1}{2}\hat{\mathcal{R}}(\hat\Delta)^\alpha{}_{\beta\hat t\hat s}\,\mathcal{U}^{\hat t}{}_{A\gamma}\,\mathcal{U}^{\hat s}_{B\delta}\,\epsilon^{AB}\,\mathbb{C}^{\delta\epsilon}\,\overline{\zeta}_\alpha\zeta_\epsilon\,\overline{\zeta}^\beta\zeta^\gamma \tag{A.2}
$$

according to the rules explained in reference [1] where $\hat{\Delta}^{\alpha}{}_{\beta}$ is the $\mathrm{Sp}(2n_H)$ connection on the quaternionic manifold $\mathcal{M}_Q$

$$\hat{\Delta}^{\alpha}_{u\beta} = \Delta^{\alpha}_{u\beta} + A^I_u \Delta^{\alpha}_{I\beta} \tag{A.3}$$

One obtains

$$\hat{\mathcal{R}}^{\alpha}{}_{\beta ts} \;=\; \mathcal{R}^{\alpha}{}_{\beta ts} + F^I_{ts}\Delta_I{}^{\alpha}{}_{\beta} - A^I_{[t}\nabla_{s]}\Delta_I{}^{\alpha}{}_{\beta} + A^I_t A^J_s\, \Delta_I{}^{\alpha}{}_{\gamma}\Delta_J{}^{\gamma}{}_{\beta} \tag{A.4}$$

where the hatted indices in the l.h.s. refer to the quaternionic manifold, $F^I_{ts} = \partial_{[t}A^I_{s]}$ and $\mathcal{R}^{\alpha}{}_{\beta ts}$ is the symplectic curvature on $\mathcal{M}_T$ defined by

$$\mathcal{R}^{\alpha}{}_{\beta} = \mathrm{d}\Delta^{\alpha}{}_{\beta} + \Delta^{\alpha}{}_{\gamma} \wedge \Delta^{\gamma}{}_{\beta}. \tag{A.5}$$

Taking into account the previous modifications $\mathcal{L}^{\mathrm{inv}}_{\mathrm{4f}}$ becomes

$$\begin{aligned}
\mathcal{L}^{\mathrm{inv}}_{\mathrm{4f}} \;=\;& \frac{\mathrm{i}}{2}\left(g_{i\bar{\jmath}}\overline{\lambda}^{iA}\gamma_{\sigma}\lambda^{\bar{\jmath}}_B - 2\delta^A_B\,\overline{\zeta}^{\alpha}\gamma_{\sigma}\zeta_{\alpha}\right)\overline{\psi}_{A|\mu}\gamma_{\lambda}\psi^B_{\nu}\,\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}} + \\
& -\frac{1}{6}\left(C_{ijk}\,\overline{\lambda}^{iA}\gamma^{\mu}\psi^B_{\mu}\,\overline{\lambda}^{jC}\lambda^{kD}\epsilon_{AC}\epsilon_{BD} + \mathrm{c.c.}\right) + \\
& -2\overline{\psi}^A_{\mu}\psi^B_{\nu}\overline{\psi}^{\mu}_A\psi^{\nu}_B + 2g_{i\bar{\jmath}}\,\overline{\lambda}^{iA}\gamma_{\mu}\psi^B_{\nu}\,\overline{\lambda}^{\bar{\imath}}_A\gamma^{\mu}\psi^{\nu}_B + \\
& +\frac{1}{4}\left(R_{i\bar{\jmath}l\bar{k}} + g_{i\bar{k}}\,g_{l\bar{\jmath}} - \frac{3}{2}g_{i\bar{\jmath}}\,g_{l\bar{k}}\right)\overline{\lambda}^{iA}\lambda^{lB}\,\overline{\lambda}^{\bar{\jmath}}_A\lambda^{\bar{k}}_B + \\
& +\frac{1}{4}g_{i\bar{\jmath}}\,\overline{\zeta}^{\alpha}\gamma_{\mu}\zeta_{\alpha}\overline{\lambda}^{iA}\gamma^{\mu}\lambda^{\bar{\jmath}}_A + \frac{1}{2}\left(\mathcal{R}^{\alpha}{}_{\beta ts} + F^I_{ts}\Delta_I{}^{\alpha}{}_{\beta} + \right. \\
& \left. -A^I_{[t}\nabla_{t]}\Delta_I{}^{\alpha}{}_{\beta} + A^I_t A^J_s\Delta_I{}^{\alpha}{}_{\gamma}\Delta_J{}^{\gamma}{}_{\beta}\right)\epsilon^{AB}\,\mathbb{C}^{\delta\eta}\,\overline{\zeta}_{\alpha}\zeta_{\eta}\,\overline{\zeta}^{\beta}\zeta^{\gamma} + \\
& -\left[\frac{\mathrm{i}}{12}\nabla_m C_{jkl}\overline{\lambda}^{jA}\lambda^{mB}\overline{\lambda}^{kC}\lambda^{lD}\epsilon_{AC}\epsilon_{BD} + \mathrm{c.c.}\right] + \\
& +g_{i\bar{\jmath}}\,\overline{\psi}^A_{\mu}\lambda^{\bar{\jmath}}_A\,\overline{\psi}^{\mu}_B\lambda^{iB} + 2\overline{\psi}^A_{\mu}\zeta^{\alpha}\,\overline{\psi}^{\mu}_A\zeta_{\alpha} + \\
& +\left(\epsilon_{AB}\,\mathbb{C}_{\alpha\beta}\,\overline{\psi}^A_{\mu}\zeta^{\alpha}\,\overline{\psi}^{B|\mu}\zeta^{\beta} + \mathrm{c.c.}\right) + \\
& +120\mathcal{M}^{IJ}\Delta_I{}^{\alpha}{}_{\beta}\,\Delta_J{}^{\gamma}{}_{\delta}\,\zeta_a\gamma^{\mu}\zeta^{\beta}\,\zeta_{\gamma}\gamma_{\mu}\zeta^{\delta} + \\
& +24\,\mathrm{i}\,\mathcal{M}^{IJ}\Delta_I{}^{\alpha}{}_{\beta}\,\zeta_{\alpha}\gamma_{\mu}\zeta^{\beta}\left[\mathcal{U}_J{}^{A\gamma}\overline{\psi}_{A|\nu}\gamma^{\mu\nu}\zeta_{\gamma} + \mathcal{U}_{J\,A\gamma}\overline{\psi}^A_{\nu}\gamma^{\mu\nu}\zeta^{\gamma}\right] + \\
& +24\,\mathrm{i}\,\mathcal{M}^{IJ}\Delta_I{}^{\alpha}{}_{\beta}\,\zeta_{\alpha}\gamma^{\mu}\zeta^{\beta}\left[\mathcal{U}_J{}^{A\gamma}\overline{\psi}_{A|\mu}\zeta_{\gamma} + \mathcal{U}_{J\,A\gamma}\overline{\psi}^A_{\mu}\zeta^{\gamma}\right].
\end{aligned} \tag{A.6}$$

The supersymmetry transformation laws of the fermions turn out to be

$$\begin{aligned}
\delta\psi_{A|\mu} &= \mathcal{D}_\mu\epsilon_A - \frac{1}{4}\left(\partial_i K\,\overline{\lambda}^{iB}\varepsilon_B - \partial_{\bar\imath}K\,\overline{\lambda}^{\bar\imath}_B\varepsilon^B\right)\psi_{A\mu} + \\
&\quad -h^I_\mu\omega_{IA}{}^B\varepsilon_B + 12\,\mathrm{i}\,\Delta^{I\alpha}{}_\beta\zeta_\alpha\gamma_\mu\zeta^\beta\omega_{IA}{}^B\varepsilon_B + \\
&\quad -2\omega_{u\,A}{}^B P^u{}_{C\alpha}\left(\epsilon^{CD}\,\mathbb{C}^{\alpha\beta}\,\overline{\zeta}_\beta\varepsilon_D + \overline{\zeta}^\alpha\epsilon^C\right)\psi_{B\mu} + \\
&\quad -2\omega_{I\,A}{}^B\mathcal{U}^I{}_{C\alpha}\left(\epsilon^{CD}\,\mathbb{C}^{\alpha\beta}\,\overline{\zeta}_\beta\varepsilon_D + \overline{\zeta}^\alpha\varepsilon^C\right)\psi_{B\mu} + \\
&\quad + \left(A_A{}^{\nu B}\eta_{\mu\nu} + A'_A{}^{\nu B}\gamma_{\mu\nu}\right)\varepsilon_B + \\
&\quad + \left[\mathrm{i}S_{AB}\eta_{\mu\nu} + \epsilon_{AB}\left(T^-_{\mu\nu} + U^+_{\mu\nu}\right)\right]\gamma^\nu\varepsilon^B \qquad \text{(A.7)}\\
\delta\lambda^{iA} &= \frac{1}{4}\left(\partial_j K\,\overline{\lambda}^{jB}\epsilon_B - \partial_{\bar\jmath}K\,\overline{\lambda}^{\bar\jmath}_B\varepsilon^B\right)\lambda^{iA} + \\
&\quad -2\omega_I{}^A{}_B\mathcal{U}^I{}_{C\alpha}\left(\epsilon^{CD}\,\mathbb{C}^{\alpha\beta}\,\overline{\zeta}_\beta\epsilon_D + \overline{\zeta}^\alpha\epsilon^C\right)\lambda^{iB} + \\
&\quad -\omega_u{}^A{}_B P^u{}_{C\alpha}\left(\epsilon^{CD}\,\mathbb{C}^{\alpha\beta}\,\overline{\zeta}_\beta\epsilon_D + \overline{\zeta}^\alpha\varepsilon^C\right)\lambda^{iB} + \\
&\quad -\Gamma^i{}_{jk}\,\overline{\lambda}^{kB}\varepsilon_B\,\lambda^{jA} + \mathrm{i}\left(\nabla_\mu z^i - \overline{\lambda}^{iA}\psi_{A|\mu}\right)\gamma^\mu\varepsilon^A + \\
&\quad +G^{-i}_{\mu\nu}\,\gamma^{\mu\nu}\varepsilon_B\epsilon^{AB} + D^{iAB}\varepsilon_B \qquad \text{(A.8)}\\
\delta\zeta_\alpha &= -\Delta_{u\,\alpha}{}^\beta P^u{}_{\gamma A}\left(\epsilon^{AB}\,\mathbb{C}^{\gamma\delta}\,\overline{\zeta}_\delta\varepsilon_B + \overline{\zeta}^\gamma\varepsilon^A\right)\zeta_\beta + \\
&\quad -h^I_\mu\omega_{IA}{}^B\varepsilon_B - 12\,\mathrm{i}\,\Delta^{I\alpha}{}_\beta\zeta_\alpha\gamma_\mu\zeta^\beta\omega_{IA}{}^B\varepsilon_B + \\
&\quad +\frac{1}{4}\left(\partial_i K\,\overline{\lambda}^{iB}\varepsilon_B - \partial_{\bar\imath}K\,\overline{\lambda}^{\bar\imath}_B\varepsilon^B\right)\zeta_\alpha + \\
&\quad +\mathrm{i}\left(P_u{}^{B\beta}\nabla_\mu q^u - \epsilon^{BC}\,\mathbb{C}^{\beta\gamma}\,\overline{\zeta}_\gamma\psi_{C|\mu} - \overline{\zeta}^\beta\psi^B_\mu\right)\gamma^\mu\varepsilon^A\epsilon_{AB}\,\mathbb{C}_{\alpha\beta} + \\
&\quad -\mathrm{i}h^I_\mu\mathcal{U}_I{}^{B\beta}\gamma^\mu\varepsilon^A\epsilon_{AB}\,\mathbb{C}_{\alpha\beta} + N^A_\alpha\varepsilon_A. \qquad \text{(A.9)}
\end{aligned}$$

Note that these transformation laws, which have been obtained from the solution of Bianchi identities in superspace (see Appendix B), could be obtained from the corresponding formulae of $N=2$ standard supergravity of reference [11] by reducing the quaternionic index as explained in reference [1] provided we make a suitable Ansatz in superspace for the 1–form h_I as explained in the next Appendix.

Appendix B: Solution of Bianchi identities and the construction of the rheonomic Lagrangian.

In this Appendix we first describe the geometric approach for the derivation of the $N = 2$ supersymmetry transformation laws of the physical fields, and then we construct the rheonomic superspace Lagrangian. The solution of Bianchi identities in superspace will provide us with the supersymmetry transformation laws in space–time, while the restriction of the rheonomic Lagrangian to space–time will give the Lagrangian of section 3 and its completion given in Appendix A. Since the present approach is completely analogous to that used for $N = 2$ standard supergravity we will be very short referring for more details to Appendices A and B of the quoted reference.
The first step to perform is to extend the physical fields to superfields in $N = 2$ superspace: that means that the space–time 2–form B_I, 1–forms ω^{ab}, V^a,ψ^A, ψ_A, A^Λ, and the space–time 0–forms λ^{iA}, $\lambda^{\bar{\imath}}_A$, z^i, $\overline{z}^{\bar{\imath}}$, ζ_α, ζ^α, q^u are promoted to 2–, 1– and 0–superforms in $N = 2$ superspace, respectively.
The definition of the superspace "curvatures" in the gravitational and vector multiplet sectors is identical to that of standard $N = 2$ supergravity except for the fact that the composite 1–form connections $\mathcal{Q}$, $\omega^A{}_B$ and $\Delta^\alpha{}_\beta$ are now restricted to $\mathcal{M}_T$ instead of the quaternionic manifold.
We have:

$$
\begin{aligned}
T^a &\equiv \mathrm{d}V^a - \omega^a{}_b \wedge V^b - \mathrm{i}\,\overline{\psi}_A \wedge \gamma^a \psi^A = 0 && \text{(A.1)}\\
\rho_A &\equiv \mathrm{d}\psi_A - \frac{1}{4}\gamma_{ab}\,\omega^{ab} \wedge \psi_A + \frac{\mathrm{i}}{2}\mathcal{Q} \wedge \psi_A + \omega_A{}^B \wedge \psi_B \equiv \nabla\psi_A && \text{(A.2)}\\
\rho^A &\equiv \mathrm{d}\psi^A - \frac{1}{4}\gamma_{ab}\,\omega^{ab} \wedge \psi^A - \frac{\mathrm{i}}{2}\mathcal{Q} \wedge \psi^A + \omega^A{}_B \wedge \psi^B \equiv \nabla\psi^A && \text{(A.3)}\\
R^{ab} &\equiv \mathrm{d}\omega^{ab} - \omega^a{}_c \wedge \omega^{cb} && \text{(A.4)}\\
\nabla z^i &= \mathrm{d}z^i && \text{(A.5)}\\
\nabla \bar{z}^{\bar{\imath}} &= \mathrm{d}\bar{z}^{\bar{\imath}} && \text{(A.6)}\\
\nabla\lambda^{iA} &\equiv \mathrm{d}\lambda^{iA} - \frac{1}{4}\gamma_{ab}\,\omega^{ab}\lambda^{iA} - \frac{\mathrm{i}}{2}\mathcal{Q}\lambda^{iA} + \Gamma^i{}_j\lambda^{jA} + \omega^A{}_B\lambda^{iB} && \text{(A.7)}\\
\nabla\lambda^{\bar{\imath}}_A &\equiv \mathrm{d}\lambda^{\bar{\imath}}_A - \frac{1}{4}\gamma_{ab}\,\omega^{ab}\lambda^{\bar{\imath}}_A + \frac{\mathrm{i}}{2}\mathcal{Q}\lambda^{\bar{\imath}}_A + \Gamma^{\bar{\imath}}{}_{\bar{\jmath}}\lambda^{\bar{\jmath}}_A + \omega_A{}^B\lambda^{\bar{\imath}}_B && \text{(A.8)}\\
F^\Lambda &\equiv \mathrm{d}A^\Lambda + \overline{L}^\Lambda\overline{\psi}_A \wedge \psi_B\epsilon^{AB} + L^\Lambda\overline{\psi}^A \wedge \psi^B\epsilon_{AB} && \text{(A.9)}
\end{aligned}
$$

where $\omega_A{}^B = \frac{\mathrm{i}}{2}\omega^x\sigma^x{}_A{}^B$ ($\omega^A{}_B = \epsilon^{AC}\epsilon_{DB}\,\omega_C{}^D$) and $\mathcal{Q}$ are connections on

the SU(2) bundle defined on $\mathcal{M}_T$ and on the U(1) bundle defined on $\mathcal{M}_{SK}$, namely $\mathcal{Q} = -\frac{\mathrm{i}}{2}(\partial_i K \mathrm{d}z^i - \partial_{\bar{\imath}} K \mathrm{d}z^{\bar{\imath}})$ with K being the Kähler potential. The index Λ runs from 0 to n_V, the index 0 referring to the graviphoton. The Levi–Civita connection on $\mathcal{M}_{SK}$ is $\Gamma^i{}_j$ and $L^\Lambda(z,\overline{z}) = e^{\frac{\mathcal{K}}{2}} X^\Lambda(z)$ is the upper half (electric) of the symplectic section of special geometry. Note that equation (A.1) is a superspace constraint.
Instead of the hypermultiplet curvatures we have now the tensor multiplet curvatures defined as

$$
\begin{aligned}
H_I &\equiv \mathrm{d}B_I - \omega_{IA}{}^B \,\overline{\psi}^A \wedge \gamma_a \psi_B \wedge V^a && \text{(A.10)}\\
P^{A\alpha} &\equiv P_u{}^{A\alpha} \mathrm{d}q^u && \text{(A.11)}\\
\nabla\zeta_\alpha &\equiv \mathrm{d}\zeta_\alpha - \frac{1}{4}\omega^{ab}\gamma_{ab}\zeta_\alpha - \frac{\mathrm{i}}{2}\mathcal{Q}\zeta_\alpha + \Delta_\alpha{}^\beta \zeta_\beta && \text{(A.12)}\\
\nabla\zeta^\alpha &\equiv \mathrm{d}\zeta^\alpha - \frac{1}{4}\omega^{ab}\gamma_{ab}\zeta^\alpha + \frac{\mathrm{i}}{2}\mathcal{Q}\zeta^\alpha + \Delta^\alpha{}_\beta \zeta^\beta. && \text{(A.13)}
\end{aligned}
$$

Applying the d operator to equations (A.1)–(A.13), one finds:

$$
\begin{aligned}
&\mathcal{D}T^a && +R^{ab}\wedge V^b - \mathrm{i}\overline{\psi}^A \wedge \gamma^a \rho_A + \mathrm{i}\overline{\rho}^A \wedge \gamma^a \psi_A = 0 && \text{(A.14)}\\
&\nabla\rho_A && +\frac{1}{4}\gamma_{ab}R^{ab}\wedge\psi_A - \frac{\mathrm{i}}{2}K\wedge\psi_A - \frac{\mathrm{i}}{2}R_A{}^B \wedge \psi_B = 0 && \text{(A.15)}\\
&\nabla\rho^A && +\frac{1}{4}\gamma_{ab}R^{ab}\wedge\psi^A + \frac{\mathrm{i}}{2}K\wedge\psi^A + \frac{\mathrm{i}}{2}R^A{}_B \wedge \psi^B = 0 && \text{(A.16)}\\
&\mathcal{D}R^{ab} && = 0 && \text{(A.17)}\\
&\nabla^2 z^i && = 0 && \text{(A.18)}\\
&\nabla^2 z^{\bar{\imath}} && = 0 && \text{(A.19)}\\
&\nabla^2\lambda^{iA} && +\frac{1}{4}\gamma_{ab}R^{ab}\lambda^{iA} + \frac{\mathrm{i}}{2}K\lambda^{iA} + R^i{}_j\lambda^{jA} - \frac{\mathrm{i}}{2}R^A{}_B\lambda^{iB} = 0 && \text{(A.20)}\\
&\nabla^2\lambda^{\bar{\imath}}_A && +\frac{1}{4}\gamma_{ab}R^{ab}\lambda^{\bar{\imath}}_A - \frac{\mathrm{i}}{2}K\lambda^{\bar{\imath}}_A + R^{\bar{\imath}}{}_{\bar{\jmath}}\lambda^{\bar{\jmath}}_A - \frac{\mathrm{i}}{2}R_A{}^B\lambda^{\bar{\imath}}_B = 0 && \text{(A.21)}\\
&\nabla\hat{F}^\Lambda && -\nabla\overline{L}^\Lambda \wedge \overline{\psi}_A \wedge \psi_B \epsilon^{AB} - \nabla L^\Lambda \wedge \overline{\psi}^A \wedge \psi^B \epsilon_{AB} \\
& && +2\overline{L}^\Lambda \overline{\psi}_A \wedge \rho_B \epsilon^{AB} + 2L^\Lambda \overline{\psi}^A \wedge \rho^B \epsilon_{AB} + \\
& && +2m^{I\Lambda}\left(H_I + \omega_{I\,A}{}^B \overline{\psi}^A \gamma_a \psi_B \wedge V^a\right) = 0 && \text{(A.22)}\\
&\nabla H_I && -\nabla\omega_{I\,A}{}^B \wedge \overline{\psi}_B \wedge \gamma_a \psi^A \wedge V^a + \omega_{I\,A}{}^B \left(\overline{\psi}_B \wedge \gamma_a \rho^A + \right.
\end{aligned}
$$

$$
\left. + \overline{\psi}^A \wedge \gamma_a \rho_B \right) \wedge V^a - \mathrm{i}\, \omega_{I\,A}{}^B \overline{\psi}_B \wedge \gamma_a \psi^A \wedge \overline{\psi}_C \wedge \gamma^a \psi^C \quad (A.23)
$$

$$
\begin{aligned}
\nabla P^{A\alpha} &= 0 && (A.24)\\
\nabla^2 \zeta_\alpha &+ \frac{1}{4} R^{ab} \gamma_{ab} \zeta_\alpha + \frac{\mathrm{i}}{2} K \zeta_\alpha + R_\alpha{}^\beta \zeta_\beta = 0 && (A.25)\\
\nabla^2 \zeta^\alpha &+ \frac{1}{4} R^{ab} \gamma_{ab} \zeta^\alpha - \frac{\mathrm{i}}{2} K \zeta^\alpha + R^\alpha{}_\beta \zeta^\beta = 0 && (A.26)
\end{aligned}
$$

where the covariant derivatives are defined by equations (A.2), (A.3), (A.7), (A.8), (A.12) and (A.13).
The solution can be obtained as follows: first of all one requires that the expansion of the curvatures along the intrinsic p–forms basis in superspace, namely: V, ψ, $V \wedge V$, $V \wedge \psi$, $\psi \wedge \psi$, $V \wedge V \wedge V$, $V^a \wedge V^b \wedge \psi$, $V^a \wedge \psi \wedge \psi$ and $\psi \wedge \psi \wedge \psi$, is given in terms only of the physical fields (rheonomy). This insures that no new degree of freedom is introduced in the theory.
Secondly one writes down such expansion in a form which is compatible with all the symmetries of the theory, that is: covariance under U(1)–Kähler and SU(2)$\otimes$Sp$(2,m)\otimes$SO(n_T), Lorentz transformations and reparametrization of the scalar manifold $\mathcal{M}_{SK} \otimes \mathcal{M}_T$. This fixes completely the Ansatz for the curvatures at least if we exclude higher derivative interactions (for a more detailed explanation the interested reader is referred to the standard reference of the geometrical approach [18] and to the Appendices A and B of reference [11]).
It is important to note that, in order to satisfy the gravitino Bianchi identity in the $\psi \wedge \psi \wedge \psi$ sector, the second term in equation (A.28), even though it is a 3–fermion term, is essential since its presence allows the cancellation of the shift term S_{AB} in the gravitino parametrization (in the usual gauged $N = 2$ standard supergravity the shift S_{AB} is instead cancelled by the additional terms in the gauged curvature R^{ab}).
The final parametrizations of the superspace curvatures, are given by:

$$
\begin{aligned}
T^a &= 0 && (A.27)\\
\rho_A &= \tilde{\rho}_{A|ab} V^a \wedge V^b - \omega_{IA}{}^B \mathcal{U}^I{}_{C\alpha} \left(\epsilon^{CD} \mathbb{C}^{\alpha\beta}\, \overline{\zeta}_\beta \psi_D + \overline{\zeta}^\alpha \psi^C \right) \wedge \psi_B + \\
&\quad - h^I_a\, \omega_{IA}{}^B \psi_B \wedge V^a + \left(A_A{}^{B|b} \eta_{ab} + A'_A{}^{B|b} \gamma_{ab} \right) \psi_B \wedge V^a \\
&\quad + \left[\mathrm{i} S_{AB} \eta_{ab} + \epsilon_{AB} (T^-_{ab} + U^+_{ab}) \right] \gamma^b \psi^B \wedge V^a && (A.28)\\
R^{ab} &= \tilde{R}^{ab}{}_{cd} V^c \wedge V^d - \mathrm{i} \left(\overline{\psi}_A \theta^{A|ab}_c + \overline{\psi}^A \theta^{ab}_{A|c} \right) \wedge V^c +
\end{aligned}
$$

$$
\begin{aligned}
& +\epsilon^{abcf}\overline{\psi}^A \wedge \gamma_f \psi_B \left(A'^B{}_{A|c} - \overline{A}'_{A|c}{}^B \right) + \\
& -2\,\mathrm{i}\,\omega_{I\,A}{}^B \overline{\psi}^A \gamma^a \psi_B \left(h^{I\,b} + 12\,\mathrm{i}\,\Delta_I{}^\alpha{}_\beta\,\overline{\zeta}_\alpha \gamma^b \zeta^\beta \right) + \\
& +\mathrm{i}\epsilon^{AB}\left(T^{+ab} + U^{-ab} \right) \overline{\psi}_A \wedge \psi_B - \mathrm{i}\epsilon_{AB}\left(T^{-ab} + U^{+ab} \right) \psi^A \wedge \psi^B \\
& -S_{AB}\overline{\psi}^A \wedge \gamma^{ab}\psi^B - \overline{S}^{AB}\overline{\psi}_A \wedge \gamma^{ab}\psi_B
\end{aligned} \tag{A.29}
$$

$$
\hat{F}^\Lambda = \tilde{\hat{F}}^\Lambda_{ab} V^a \wedge V^b + \mathrm{i}\left(f_i^\Lambda \overline{\lambda}^{iA}\gamma_a \psi^B \epsilon_{AB} + \mathrm{i}\overline{f}_{\bar\imath}^\Lambda \overline{\lambda}^{\bar\imath}_A \gamma_a \psi_B \epsilon^{AB} \right) \wedge V^a \tag{A.30}
$$

$$
\nabla\lambda^{iA} = \tilde{\nabla_a\lambda}^{iA} V^a + \mathrm{i}\,\tilde{Z}^i_a \gamma^a \psi^A + G^{-i}_{ab}\gamma^{ab}\epsilon^{AB}\psi_B + W^{iAB}\psi_B \tag{A.31}
$$

$$
\nabla z^i = \tilde{Z}^i_a V^a + \overline{\lambda}^{iA}\psi_A \tag{A.32}
$$

$$
P^{A\alpha} = \tilde{P}_a{}^{A\alpha} V^a + \overline{\psi}^A \zeta^\alpha + \epsilon^{AB}\mathbb{C}^{\alpha\beta}\,\overline{\psi}_B \zeta_\beta \tag{A.33}
$$

$$
H_I = \tilde{H}_{Iabc} V^a \wedge V^b \wedge V^c - \frac{\mathrm{i}}{2}\left(\mathcal{U}_I{}^{A\alpha}\overline{\psi}_A \gamma_{ab}\zeta_\alpha - \mathcal{U}_{IA\alpha}\overline{\psi}^A \gamma_{ab}\zeta^\alpha \right) \wedge V^a \wedge V^b \tag{A.34}
$$

$$
\nabla\zeta_\alpha = \tilde{\nabla_a\zeta}_\alpha V^a + \mathrm{i}P_{a\,A\alpha}\gamma^a\psi^A - \mathrm{i}h^I_a \mathcal{U}_{I\,A\alpha}\,\gamma^a\psi^A + N^A_\alpha \psi_A \tag{A.35}
$$

where:

$$
A_A{}^{|aB} = -\frac{\mathrm{i}}{4}\,g_{\bar k l}\left(\overline{\lambda}^{\bar k}_A \gamma^a \lambda^{lB} - \delta^B_A \overline{\lambda}^{\bar k}_C \gamma^a \lambda^{lC} \right) \tag{A.36}
$$

$$
A'_A{}^{|aB} = \frac{\mathrm{i}}{4}\,g_{\bar k l}\left(\overline{\lambda}^{\bar k}_A \gamma^a \lambda^{lB} - \frac{1}{2}\delta^B_A \overline{\lambda}^{\bar k}_C \gamma^a \lambda^{lC} \right) - \frac{\mathrm{i}}{4}\delta^B_A \overline{\zeta}_\alpha \gamma^a \zeta^\alpha \tag{A.37}
$$

$$
\theta^{ab|c}_A = 2\gamma^{[a}\rho^{b]c}_A + \gamma^c\rho^{ab}_A; \quad \theta^{ab\,A}_c = 2\gamma^{[a}\rho^{b]c|A} + \gamma^c\rho^{ab|A} \tag{A.38}
$$

$$
\begin{aligned}
T^-_{ab} = {} & \left(\mathcal{N} - \overline{\mathcal{N}} \right)_{\Lambda\Sigma} L^\Sigma \left(\tilde{\hat{F}}^{\Lambda -}_{ab} + \frac{1}{8}\nabla_i f^\Lambda_j \overline{\lambda}^{iA}\gamma_{ab}\lambda^{jB}\epsilon_{AB} \right. \\
& \left. -\frac{1}{4}\mathbb{C}^{\alpha\beta}\overline{\zeta}_\alpha \gamma_{ab}\zeta_\beta L^\Lambda \right)
\end{aligned} \tag{A.39}
$$

$$
U^-_{ab} = \frac{\mathrm{i}}{4}\lambda\mathbb{C}^{\alpha\beta}\overline{\zeta}_\alpha \gamma_{ab}\zeta_\beta \tag{A.40}
$$

$$
\begin{aligned}
G^{i-}_{ab} = {} & \frac{\mathrm{i}}{2} g^{i\bar\jmath}\overline{f}^\Gamma_{\bar\jmath}\left(\mathcal{N} - \overline{\mathcal{N}} \right)_{\Gamma\Lambda} \left(\tilde{\hat{F}}^{\Lambda -}_{ab} + \frac{1}{8}\nabla_k f^\Lambda_l \overline{\lambda}^{kA}\gamma_{ab}\lambda^{lB}\epsilon_{AB} \right. \\
& \left. -\frac{1}{4}\mathbb{C}^{\alpha\beta}\overline{\zeta}_\alpha \gamma_{ab}\zeta_\beta L^\Lambda \right)
\end{aligned} \tag{A.41}
$$

$$
S_{AB} = \frac{\mathrm{i}}{2}\sigma^x{}_{AB}\omega^x_I \left(e^I_\Lambda L^\Lambda - m^{I\Lambda}M_\Lambda \right) \tag{A.42}
$$

$$
N^A_\alpha = -2\,\mathcal{U}_I{}^A{}_\alpha \left(e^I_\Lambda L^\Lambda - m^{I\Lambda}M_\Lambda \right) \tag{A.43}
$$

$$
\begin{aligned}
W^{iAB} &= \mathrm{i}\, g^{i\bar{\jmath}}\,\sigma^{x\,AB} g \mathcal{P}^x_\Lambda \overline{f}^\Lambda_{\bar{\jmath}} + \\
&\quad + \mathrm{i}\, g^{i\bar{\jmath}}\,\sigma^{x\,AB}\omega^x_I \left(e^I_\Lambda \overline{f}^\Lambda_{\bar{\jmath}} - m^{I\Lambda}\overline{h}_{\Lambda\bar{\jmath}} \right)
\end{aligned} \tag{A.44}
$$

where according to what has been discussed in the main text, $\hat{F}^\Lambda = F^\Lambda + 2m^{I\Lambda}B_I$.
Let us now discuss the appearance in the parametrization of the fermions of the shift terms S_{AB}, W^{iAB} and N^A_α.
If we set these shifts to zero, we have the solution of the Bianchi identities provided $\hat{\mathcal{F}}$ is replaced by $\mathcal{F}$. The extended solution containing these shifts can be retrieved as follows: if we look at the Bianchi identity (A.22), with $\hat{\mathcal{F}}$ replaced by $\mathcal{F}$, then adding the so far unknown shifts gives in the sector $\overline{\psi}^B \gamma^a \psi_C V^b$ the following extra terms:

$$
\left(-\frac{1}{2}\epsilon_{AB} W^{iAC} f^\Lambda_i + L^\Lambda S^{AC}\epsilon_{BA} + \text{c.c.} \right) 2\,\mathrm{i}\, \overline{\psi}^B \wedge \gamma_a \psi_C \wedge V^a = 0. \tag{A.45}
$$

Taking into account the identities of Special Geometry [11], the solution of equation (A.45) must have the following form:

$$
S^{AC} = D^{AC}_\Lambda L^\Lambda \tag{A.46}
$$

$$
W^{iAC} = 2 g^{i\bar{\jmath}} f^\Lambda_{\bar{\jmath}} D^{AC}_\Lambda \tag{A.47}
$$

where D^{AC}_Λ is a set of $n_V + 1$ SU(2) symmetric tensors so far undetermined. The magnetic mass–shifts can be obtained as follows; we set:

$$
D^{AC}_\Lambda = -\frac{\mathrm{i}}{2}\sigma^{x\,AC}\omega^x_I m^{I\Lambda} M_\Lambda \tag{A.48}
$$

then using the Special Geometry identity:

$$
g^{\bar{\imath}j} f^\Lambda_{\bar{\imath}} h_{\Sigma j} - \text{c.c.} = \mathrm{i}\delta^\Lambda_\Sigma + \overline{L}^\Lambda M_\Sigma - L^\Lambda \overline{M}_\Sigma \tag{A.49}
$$

one sees that the residual term

$$
- m^{I\Lambda} M_\Lambda \omega_{IA}{}^B \overline{\psi}^A \wedge \gamma_a \psi_B \wedge V^a \tag{A.50}
$$

is cancelled by the redefinition (2.11) using equation (A.10). This solution for the magnetic shifts can be further generalized to include electric mass–shifts by extending the definition of D^{AC}_Λ as follows:

$$D_\Lambda^{AC} \to D_\Lambda^{AC} + \frac{\mathrm{i}}{2}\sigma^{x\,AC}\omega_I^x e_\Lambda^I L^\Lambda. \qquad \text{(A.51)}$$

Indeed the new terms cancel identically by virtue of the Special Geometry formula

$$g^{\bar{\imath}j} f_{\bar{\imath}}^\Lambda f_j^\Sigma = -\frac{1}{2}\left(\mathrm{Im}\mathcal{N}\right)^{-1|\Lambda\Sigma} - L^\Lambda \overline{L}^\Sigma. \qquad \text{(A.52)}$$

Finally the mass–shift N_α^A can be computed from the gravitino Bianchi identity looking at the coefficients of the $\psi \wedge \psi \wedge \psi$ terms. Indeed in this case one finds, after a Fierz rearrangement, the following equation:

$$S_{AB} = -\frac{\mathrm{i}}{4}\sigma^x{}_{AB}\omega_I^x \mathcal{U}^{IC}{}_\alpha N_C^\alpha \qquad \text{(A.53)}$$

which determines N_A^α to be

$$N_A^\alpha = -2\mathcal{U}_{IA}{}^\alpha \left(e_\Lambda^I L^\Lambda - m^{I\Lambda} M_\Lambda\right). \qquad \text{(A.54)}$$

where we have used the identity [1]:

$$\mathcal{U}_{IA}{}^\alpha \mathcal{U}_{JB\alpha} + \mathcal{U}_{JA}{}^\alpha \mathcal{U}_{IB\alpha} = \mathcal{M}_{IJ}\epsilon_{AB}. \qquad \text{(A.55)}$$

In this way equations (A.42), (A.44) and (A.43) are reproduced.
It is important to stress that the field–strengths $\tilde{R}^{ab}{}_{cd}$, $\tilde{\rho}_{A|ab}$, $\tilde{F}_{ab}^\Lambda$, $\tilde{P}_a^{A\alpha} \equiv P_u^{A\alpha}\tilde{\nabla_a q^u}$, $\tilde{\nabla_a \lambda^{iA}}$, $\tilde{\nabla}_a\zeta_\alpha$ and their hermitian conjugates are not space–time field–strengths since they are components along the bosonic vielbeins $V^a = V_\mu^a\, dx^\mu + V_\alpha^a \mathrm{d}\theta^\alpha$ where $(V_\mu^a,\, V_\alpha^a)$ is a submatrix of the super–vielbein matrix $E^I \equiv (V^a, \psi)$ (see Appendix A of reference [11]). Note that in the component approach the "tilded" field–strengths defined in the previous equations are usually referred to as the "supercovariant" field–strengths.
The previous formulae refer to the theory without gauging of the group $\mathcal{G}$. If the gauging is turned on then we must use gauged quantities according to the rules explained in reference [11]. In particular the differentials on the scalar fields z^i, $\overline{z}^{\bar{\imath}}$ and q^u are redefined as in equation (3.35), (3.36), (3.36) and the composite connections and curvatures acquire extra terms as explained in reference [11]. Furthermore for the gauged theory the index Λ has to be split according to the discussion of subsection 3.1. For example, equation (A.24), (A.18) and (A.19) become:

$$\nabla P^{A\alpha} \quad -\left(F^{\check\Lambda} - \overline{L}^{\check\Lambda}\overline{\psi}_A\psi_B\epsilon^{AB} - L^{\check\Lambda}\overline{\psi}^A\psi^B\epsilon_{AB}\right)k^u_{\check\Lambda}P_u{}^{A\alpha} = 0 \quad \text{(A.56)}$$

$$\nabla^2 z^i \quad -\left(F^{\check\Lambda} - \overline{L}^{\check\Lambda}\overline{\psi}_A\wedge\psi_B\epsilon^{AB} - L^{\check\Lambda}\overline{\psi}^A\wedge\psi^B\epsilon_{AB}\right)k^i_{\check\Lambda} = 0 \quad \text{(A.57)}$$

$$\nabla^2 z^{\bar\imath} \quad -\left(F^{\check\Lambda} - \overline{L}^{\check\Lambda}\overline{\psi}_A\wedge\psi_B\epsilon^{AB} - L^{\check\Lambda}\overline{\psi}^A\wedge\psi^B\epsilon_{AB}\right)k^{\bar\imath}_{\check\Lambda} = 0. \quad \text{(A.58)}$$

Furthermore the curvature 2–forms K, R^{ab}, $R^{\alpha\beta}$ become gauged according to the procedure of reference [11].
Finally we recall that the solution we found is an on–shell solution, and it also determines the geometry of the Special Kähler manifold (see Appendix A of [11]) and the geometry of $\mathcal{M}_T$ that has been analyzed in reference [1].
The determination of the superspace curvatures enables us to write down the $N=2$ supersymmetry transformation laws. Indeed we recall that from the superspace point of view a supersymmetry transformation is a Lie derivative along the tangent vector:

$$\epsilon = \overline{\epsilon}^A\vec{D}_A + \overline{\epsilon}_A\vec{D}^A \tag{A.59}$$

where the basis tangent vectors $\vec{D}_A$, $\vec{D}^A$ are dual to the gravitino 1–forms:

$$\vec{D}_A\left(\psi^B\right) = \vec{D}^A\left(\psi_B\right) = \mathbb{1} \tag{A.60}$$

where $\mathbb{1}$ is the unit in spinor space.
Denoting by $\mu^{\mathcal{I}}$ and $R^{\mathcal{I}}$ the set of 1– and 2–forms $(V^a,\ \psi_A,\ \psi^A,\ A^\Lambda,\ B_I)$ and of the 2– and 3–forms $\left(R^{ab},\ \rho_A,\ \rho^A,\ F^\Lambda,\ H_I\right)$ respectively, one has:

$$\ell\mu^{\mathcal{I}} = \left(i_\epsilon \mathrm{d} + \mathrm{d} i_\epsilon\right)\mu^{\mathcal{I}} \equiv (D\epsilon)^{\mathcal{I}} + i_\epsilon R^{\mathcal{I}} \tag{A.61}$$

where D is the derivative covariant with respect to the $N=2$ Poincaré superalgebra and i_ϵ is the contraction operator along the tangent vector ϵ.
In our case:

$$(D\epsilon)^a = \mathrm{i}\left(\overline{\psi}_A\gamma^a\epsilon^A + \overline{\psi}^A\gamma^a\epsilon_A\right) \tag{A.62}$$

$$(D\epsilon)^\alpha = \nabla\epsilon^\alpha \tag{A.63}$$

$$(D\epsilon)^\Lambda = 2\overline{L}^\Lambda\overline{\psi}_A\epsilon_B\epsilon^{AB} + 2L^\Lambda\overline{\psi}^A\epsilon^B\epsilon_{AB} \tag{A.64}$$

$$(D\epsilon)_{\mathcal{I}} = \omega_{I\,A}{}^B\left(\psi_B\gamma_a\epsilon^A + \overline{\psi}^A\gamma_a\epsilon_B\right)V^a \tag{A.65}$$

(here α is a spinor index)
For the 0–forms which we denote shortly as $\nu^{\mathcal{I}} \equiv (q^u, z^i, \overline{z}^{\overline{\imath}}, \lambda^{iA}, \lambda^{\overline{\imath}}_A, \zeta_\alpha, \zeta^\alpha$ we have the simpler result:

$$\ell_\epsilon = i_\epsilon \mathrm{d}\nu^{\mathcal{I}} = i_\epsilon \left(\nabla \nu^{\mathcal{I}} - \text{connection terms.}\right) \tag{A.66}$$

Using the parametrizations given for $R^{\mathcal{I}}$ and $\nabla \nu^{\mathcal{I}}$ and identifying δ_ϵ with the restriction of ℓ_ϵ to space–time it is immediate to find the $N = 2$ supersymmetry laws for all the fields. The explicit formulae are given in Appendix A.

Let us now derive the space–time Lagrangian from the rheonomic Lagrangian. The meaning of the rheonomic Lagrangian has been discussed in detail in the literature. In our case one can repeat almost verbatim, apart of some minor modifications, the considerations given in Appendix B of [11]. We limit ourselves to write down the rheonomic Lagrangian up to 4–fermion terms since the 4–fermion terms are almost immediately reconstructed from their space–time expression given in Appendix A by promoting the various terms to 4–forms in superspace. The most general Lagrangian (up to 4–fermion terms) has the following form (wedge products are omitted):

$$\mathcal{A} = \int_{\mathcal{M}^4 \subset \mathcal{M}^{4|8}} \mathcal{L}. \tag{A.67}$$

$$\begin{aligned}
\mathcal{L}_{\text{Grav}} &= R^{ab} V^c V^d \epsilon_{abcd} - 4\left(\overline{\psi}^A \gamma_a \rho_A - \overline{\psi}_A \gamma_a \rho^A\right) V^a \qquad \text{(A.68)} \\
\mathcal{L}_{\text{kin}} &= \beta_1 g_{i\overline{\jmath}} \left[\mathbf{Z}^i_a \left(\nabla \overline{z}^{\overline{\jmath}} - \overline{\psi}^A \lambda^{\overline{\jmath}}_A\right) + \overline{\mathbf{Z}}^{\overline{\jmath}}_a \left(\nabla z^i - \overline{\psi}_A \lambda^{iA}\right)\right] \Omega^a + \\
&\quad -\frac{1}{4}\beta_1 g_{i\overline{\jmath}} \mathbf{Z}^i_e \overline{\mathbf{Z}}^{\overline{\jmath}}_f \eta^{ef} \Omega + \\
&\quad +\mathrm{i}\beta_2 g_{i\overline{\jmath}} \left(\overline{\lambda}^{iA} \gamma^a \nabla \lambda^{\overline{\jmath}}_A + \overline{\lambda}^{\overline{\jmath}}_A \gamma^a \nabla \lambda^{iA}\right) \Omega_a + \\
&\quad +\mathrm{i}\beta_3 \left(\mathcal{N}_{\Lambda\Sigma} \hat{\mathbf{F}}^{\Lambda+}_{ab} + \overline{\mathcal{N}}_{\Lambda\Sigma} \hat{\mathbf{F}}^{\Lambda-}_{ab}\right) \left[\hat{F}^\Sigma + \right. \\
&\qquad \left. -\mathrm{i}\left(f^\Sigma_i \overline{\lambda}^{iA} \gamma_c \psi^B \epsilon_{AB} + \overline{f}^\Sigma_{\overline{\imath}} \overline{\lambda}^{\overline{\imath}}_A \gamma_c \psi_B \epsilon^{AB}\right) V^c\right] V^a V^b + \\
&\quad -\frac{1}{24}\beta_3 \left(\overline{\mathcal{N}}_{\Lambda\Sigma} \hat{\mathbf{F}}^{\Lambda+}_{ef} \hat{\mathbf{F}}^{\Sigma+\, ef} - \mathcal{N}_{\Lambda\Sigma} \hat{\mathbf{F}}^{\Lambda-}_{ef} \hat{\mathbf{F}}^{\Lambda-\, ef}\right) \Omega + \\
&\quad +d_1 M^{IJ} \mathbf{h}_{Id} \\
&\qquad \left[H_J - 2\,\mathrm{i}\,\Delta_J{}^\alpha{}_\beta\, \overline{\zeta}_\alpha \gamma^a \zeta^\beta \Omega_a\right.
\end{aligned}$$

$$
\begin{aligned}
& \qquad -\frac{\mathrm{i}}{2}\left(g_J{}^{A\alpha}\overline{\psi}_A\gamma_{ab}\zeta_\alpha - g_{JA\alpha}\overline{\psi}^A\gamma_{ab}\zeta^\alpha\right)V^aV^b\Big]V^d + \\
& -\frac{1}{48}d_1 M^{IJ}\mathbf{h}_{Ie}\mathbf{h}_{Jf}\eta^{ef}\Omega + \\
& +b_1\mathbf{P}_{aA\alpha}\left(P^{A\alpha} - \overline{\psi}^A\zeta^\alpha - \epsilon^{AB}\mathbb{C}^{\alpha\beta}\overline{\psi}_B\zeta_\beta\right)\Omega^a + \\
& -\frac{1}{8}b_1\mathbf{P}_e{}^{A\alpha}\mathbf{P}^e{}_{A\alpha}\Omega + \\
& +\mathrm{i}b_2\left(\overline{\zeta}^\alpha\gamma^a\nabla\zeta_\alpha + \overline{\zeta}_\alpha\gamma^a\nabla\zeta^\alpha\right)\Omega_a + \\
& +d_2\big(H_I - 2\,\mathrm{i}\,\Delta_I{}^\alpha{}_\beta\,\overline{\zeta}_\alpha\gamma^a\zeta^\beta\Omega_a + \\
& \qquad +\omega_{IA}{}^B\,\overline{\psi}^A\gamma_a\psi_BV^a\big)\,A^I{}_uP^u{}_{C\alpha}P^{C\alpha} \qquad (A.69)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Pauli}} &= \beta_5\hat{F}^\Lambda\left(\mathcal{N}_{\Lambda\Sigma}L^\Sigma\overline{\psi}^A\psi^B\epsilon_{AB} + \overline{\mathcal{N}}_{\Lambda\Sigma}\overline{L}^\Sigma\overline{\psi}_A\psi_B\epsilon^{AB}\right) + \\
& +\mathrm{i}\beta_6\hat{F}^\Lambda\left(\overline{\mathcal{N}}_{\Lambda\Sigma}f_i^\Sigma\overline{\lambda}^{iA}\gamma_a\psi^B\epsilon_{AB} + \mathcal{N}_{\Lambda\Sigma}\overline{f}_{\bar{\imath}}^\Sigma\overline{\lambda}_A^{\bar{\imath}}\gamma_a\psi_B\epsilon^{AB}\right)V^a + \\
& +\beta_7\hat{F}^\Lambda\left(\mathcal{N} - \overline{\mathcal{N}}\right)_{\Lambda\Sigma} \\
& \quad \left(\nabla_i f_j^\Sigma\overline{\lambda}^{iA}\gamma_{ab}\lambda^{jB}\epsilon_{AB} - \nabla_{\bar{\imath}}\overline{f}_{\bar{\jmath}}^\Sigma\overline{\lambda}_A^{\bar{\imath}}\gamma_{ab}\lambda_B^{\bar{\jmath}}\epsilon^{AB}\right)V^aV^b + \\
& +b_5\hat{F}^\Lambda\left(\mathcal{N} - \overline{\mathcal{N}}\right)_{\Lambda\Sigma}\left(L^\Sigma\overline{\zeta}_\alpha\gamma_{ab}\zeta_\beta\mathbb{C}^{\alpha\beta} - \overline{L}^\Sigma\overline{\zeta}^\alpha\gamma_{ab}\zeta^\beta\mathbb{C}_{\alpha\beta}\right)V^aV^b + \\
& +b_3\left(P^{A\alpha}\overline{\zeta}_\alpha\gamma^{ab}\psi_A + P_{A\alpha}\overline{\zeta}^\alpha\gamma^{ab}\psi^A\right)\epsilon_{abcd}V^cV^d + \\
& +d_3\left(H_I - 2\,\mathrm{i}\,\Delta_I{}^\alpha{}_\beta\overline{\zeta}_\alpha\gamma^a\zeta^\beta\Omega_a\right)\left(\mathcal{U}^{IA\gamma}\overline{\zeta}_\gamma\psi_A + \mathcal{U}^I{}_{A\gamma}\overline{\zeta}^\gamma\psi^A\right) + \\
& +d_4\left(H_I - 2\mathrm{i}\Delta_I{}^\alpha{}_\beta\overline{\zeta}_\alpha\gamma^a\zeta^\beta\Omega_a\right)\Delta^I{}_\gamma{}^\delta\left(\overline{\zeta}^\gamma\gamma_d\zeta_\delta\right)V^d \qquad (A.70)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{top}} &= \alpha\, e_\Lambda^I\Big[\hat{F}^\Lambda - L^\Lambda\overline{\psi}^A\psi^B\epsilon_{AB} - \overline{L}^\Lambda\overline{\psi}_A\psi_B\epsilon^{AB} - \mathrm{i}\big(f_i^\Lambda\lambda^{iA}\gamma_a\psi^B\epsilon_{AB} + \\
& \qquad +\overline{f}_{\bar{\imath}}^\Lambda\lambda_A^{\bar{\imath}}\gamma_a\psi_B\epsilon^{AB}\big)V^a - m^{J\Lambda}B_J\Big]B_I \qquad (A.71)
\end{aligned}
$$

$$
\mathcal{L}_{\mathrm{tors}} = \left(\beta_4 g_{i\bar{\jmath}}\overline{\lambda}^{iA}\gamma_a\lambda_A^{\bar{\jmath}} + b_4\overline{\zeta}^\alpha\gamma_a\zeta_\alpha\right)T_bV^bV^a \qquad (A.72)
$$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Shifts}} &= \mathrm{i}\delta_1\left(S_{AB}\,\overline{\psi}^A\gamma_{ab}\psi^B + \overline{S}^{AB}\,\overline{\psi}_A\gamma_{ab}\psi_B\right)V^aV^b + \\
& +\mathrm{i}\,\delta_2\,g_{i\bar{\jmath}}\left(W^{iAB}\,\overline{\lambda}_A^{\bar{\jmath}}\gamma^a\psi_B + W^{\bar{\jmath}AB}\,\overline{\lambda}^{iA}\gamma^a\psi^B\right)\Omega_a + \\
& +\mathrm{i}\,\delta_3\left(N_A^\alpha\,\overline{\zeta}_\alpha\gamma^a\psi^A + N_\alpha^A\,\overline{\zeta}^\alpha\gamma^a\psi_A\right)\Omega_a + \\
& +\big(\delta_4\nabla_u N_A^\alpha P^u{}_{B\beta}\epsilon^{AB}\mathbb{C}^{\alpha\beta}\overline{\zeta}_\alpha\zeta_\gamma + \delta_5\nabla_i N_A^\alpha\overline{\zeta}_\alpha\lambda^{iA} +
\end{aligned}
$$

$$+\delta_6 g_{i\bar{\jmath}} \nabla_k W^{\bar{\jmath}}_{AB} \overline{\lambda}^{iA} \lambda^{kB} + \text{c.c.} \Big) \, \Omega \tag{A.73}$$

$$\mathcal{L}_{\text{Potential}} \;=\; -\frac{1}{6} \mathcal{V}(q, z, \overline{z}) \Omega \tag{A.74}$$

where we have defined

$$\Omega = \epsilon_{abcd} V^a V^b V^c V^d, \quad \Omega_a = \epsilon_{abcd} V^b V^c V^d \tag{A.75}$$

and the scalar potential $\mathcal{V}(q, z, \overline{z})$ is given by equation (3.39). Furthermore we have introduced auxiliary 0–forms $\hat{\mathbf{F}}^{\pm\Lambda}_{ab}$, $\mathbf{Z}^i_a$, $\mathbf{h}_{Ia}$, $\mathbf{P}_a{}^{A\alpha}$ whose variational equations identify them with $\tilde{\hat{F}}{}^{\pm\Lambda}_{ab}$, $\tilde{Z}^i_a$, $\tilde{h}_{Ia}$, $\tilde{P}_a{}^{A\alpha}$ defined by the solution of the Bianchi identities. These auxiliary fields have to be introduced in the kinetic terms of the Lagrangian in order to avoid the Hodge duality operator which would destroy the independence of the variational equation from the particular bosonic hypersurface of integration.
The variational equations, together with the principles of rheonomy fix the undetermined coefficients in the Lagrangian to the following values:

$$\begin{aligned}
&\beta_1 = \frac{2}{3}; \quad \beta_2 = -\frac{1}{3}; \quad \beta_3 = 4\mathrm{i}; \quad \beta_4 = -1; \quad \beta_5 = 4; \quad \beta_6 = -4; \\
&\beta_7 = \frac{1}{2}; \quad b_1 = -\frac{4}{3}; \quad b_2 = \frac{2}{3}; \quad b_3 = 2; \quad b_4 = -2; \quad b_5 = 1; \\
&\delta_1 = 4; \quad \delta_2 = \frac{2}{3}; \quad \delta_3 = -\frac{4}{3}; \quad \delta_4 = -\frac{1}{12}; \quad \delta_5 = -\frac{1}{3}; \quad \delta_6 = \frac{1}{18}; \\
&d_1 = -8; \quad d_2 = 8; \quad d_3 = -8; \quad d_4 = -8\mathrm{i}; \quad \alpha = 8.
\end{aligned} \tag{A.76}$$

In order to obtain the space–time Lagrangian the last step to perform is the restriction of the 4–form Lagrangian from superspace to space–time. Namely we restrict all the terms to the $\theta = 0$, $\mathrm{d}\theta = 0$ hypersurface $\mathcal{M}^4$. In practice one first goes to the second order formalism by identifying the auxiliary 0–form fields as explained before. Then one expands all the forms along the $\mathrm{d}x^\mu$ differentials and restricts the superfields to their lowest ($\theta = 0$) component. Finally the coefficient of:

$$dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma \;=\; \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{g}} \left(\sqrt{g} \, \mathrm{d}^4 x \right) \tag{A.77}$$

gives the Lagrangian density written in section 3. The overall normalization of the space–time action has been chosen such as to be the standard one for the Einstein term.

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